Using Electron Scattering Data to Model Neutrino Scattering on Nuclear Targets

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BNL Seminar

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Based on work with Brian Coopersmith (Rochester)

And M. Eric Christy (Hampton)

With thanks to **Philip A. Rodrigues** for providing results from K2K NEUT and **Tomasz Golan** for providing results form NUWRO (http://borg.ift.uni.wroc.pl/nuwro/)

Philosophy

- Models should be able to predict W1, W2, W3 for electron scattering, charged current and neutral current processes. With these, the energy and y dependence of all predictions are well determined for all the processes.

- Models should be tested against electron scattering data to make sure that the vector contribution is properly modeled.

- Neutrino data should only be used to test the axial contribution and axial-vector interference in the model.

- In this talk I focus on how to use parameters extracted from electron scattering data to describe neutrino scattering cross section.

Topics to be covered

1. Shape of Quasielastic Peak – "effective spectral functions" (new results Feb. 2014)
2. Transverse Enhancement in Quasielastic Scattering

I will say a few words on the axial form factors

I have no time to cover Inelastic scattering, the Bodek-Yang model.

Longitudinal vs Transverse
(Scattering from Quarks, structure function description)

- Transverse means polarization of the electric field perpendicular to the direction of motion.   i.e. the spin/helicity is along the the direction of motion (e.g. real photons).
- Since quarks are very low mass, they have +-helicity. Therefore, the absorption of transverse photons results in spin/helicity flip of the quarks (which is the dominant process at large Q2.

- Longitudinal means that electric field is parallel the direction of motion. This is only possible for off-shell virtual photons. This means that the photons can only be absorbed by quarks which have a transverse momentum (e.g. from gluon emission, or binding the nucleon, target mass).

- For charged current scattering from individual quarks: vector=axial

2 Electron-nucleon and muon-nucleon scattering

Structure function description

$$\frac{d^2\sigma}{d\Omega dE'}(E_0, E', \theta) = \frac{4\alpha^2 E'^2}{Q^4}\cos^2(\theta/2)$$

$$\times \left[\mathcal{F}_2(x, Q^2)/\nu + 2\tan^2(\theta/2)\mathcal{F}_1(x, Q^2)/M\right]$$

$$\frac{d^2\sigma}{d\Omega dE'} = \Gamma\left[\sigma_T(x, Q^2) + \epsilon\sigma_L(x, Q^2)\right]$$

Transverse and longitudinal description

$$\Gamma = \frac{\alpha K E'}{4\pi^2 Q^2 E_0}\left(\frac{2}{1-\epsilon}\right)$$

$$\epsilon = \left[1 + 2(1 + \frac{Q^2}{4M^2x^2})tan^2\frac{\theta}{2}\right]^{-1}$$

$$K = \frac{Q^2(1-x)}{2Mx}.$$

Virtual boson polarization is a function of y (or angle)

Ratio of L/T

$$\mathcal{R}(x, Q^2) = \frac{\sigma_L}{\sigma_T} = \frac{\mathcal{F}_2}{2x\mathcal{F}_1}(1 + \frac{4M^2x^2}{Q^2}) - 1 = \frac{\mathcal{F}_L}{2x\mathcal{F}_1}$$

terms of the structure functions $\mathcal{F}_1 = MW_1(x, Q^2)$, $\mathcal{F}_2 = \nu W_2(x, Q^2)$ and $\mathcal{F}_3 = \nu W_3(x, Q^2)$:

Relation between the two descriptions.

$$\mathcal{F}_1 = \frac{MK}{4\pi^2\alpha}\sigma_T,$$

Purely transverse

$$\mathcal{F}_2 = \frac{\nu K(\sigma_L + \sigma_T)}{4\pi^2\alpha(1 + \frac{Q^2}{4M^2x^2})}$$

Sum of T and L

Since R is small at high Q^2, we use mixed description in terms of F2, R, F3 (shown below for neutrino scattering)

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx\,dy} = \frac{G_F^2 M E_\nu}{\pi}\left(\left[1 - y\left(1 + \frac{Mx}{2E_\nu}\right) + \frac{y^2}{2}\left(\frac{1 + (\frac{2Mx}{Q})^2}{1+R}\right)\right]\mathcal{F}_2 \pm \left[y - \frac{y^2}{2}\right]x\mathcal{F}_3\right)$$

For neutrinos, the structure functions have both vector and axial components)

Relations between electric and magnetic form factors and structure functions for elastic electron scattering on free nucleons

$$\mathcal{W}_{1p}^{elastic} = \delta(\nu - \frac{Q^2}{2M})\tau|G_{Mp}(Q^2)|^2$$

$$\mathcal{W}_{1n}^{elastic} = \delta(\nu - \frac{Q^2}{2M})\tau|G_{Mn}(Q^2)|^2$$

and

$$\boxed{\mathcal{W}_{2p}^{elastic} = \delta(\nu - \frac{Q^2}{2M})\frac{[G_{Ep}(Q^2)]^2 + \tau[G_{Mp}(Q^2)]^2}{1+\tau}}$$

$$\mathcal{W}_{2n}^{elastic} = \delta(\nu - \frac{Q^2}{2M})\frac{[G_{En}(Q^2)]^2 + \tau[G_{Mn}(Q^2)]^2}{1+\tau}$$

$$R_{p,n}^{elastic}(x=1, Q^2) = \frac{\sigma_L^{elastic}}{\sigma_T^{elastic}} = \frac{4M^2}{Q^2}\left(\frac{G_E^2}{G_M^2}\right)$$

Here, $\tau = Q^2/4M_{p,n}^2$, where $M_{p,n}$ are the masses of proton and neutron. Therefore, G_{Mp} and G_{Mn} contribute to the transverse virtual photo-absorption cross section, and G_{Ep} and G_{En} contribute to the longitudinal cross section.

$$\boxed{\mathcal{F}_A \approx -1.267/(1+Q^2/M_A^2)^2}$$

For Neutrino QE scattering: Vector form factors are known from electron scattering. But we also have axial form factors

$$W_{1\text{-Qelastic}}^{\nu\text{-vector}} = \delta\left(\nu - \frac{Q^2}{2M}\right)\tau|\mathcal{G}_M^V(Q^2)|^2,$$

$$W_{1\text{-Qelastic}}^{\nu\text{-axial}} = \delta\left(\nu - \frac{Q^2}{2M}\right)(1+\tau)|\mathcal{F}_A(Q^2)|^2,$$

$$\boxed{W_{2\text{-Qelastic}}^{\nu\text{-vector}} = \delta\left(\nu - \frac{Q^2}{2M}\right)|\mathcal{F}_V(Q^2)|^2,}$$

$$\boxed{W_{2\text{-Qelastic}}^{\nu\text{-axial}} = \delta\left(\nu - \frac{Q^2}{2M}\right)|\mathcal{F}_A(Q^2)|^2,}$$

$$W_{3\text{-Qelastic}}^{\nu} = \delta\left(\nu - \frac{Q^2}{2M}\right)|2\mathcal{G}_M^V(Q^2)\mathcal{F}_A(Q^2)|,$$

where

$$\mathcal{G}_E^V(Q^2) = G_E^p(Q^2) - G_E^n(Q^2),$$

$$\mathcal{G}_M^V(Q^2) = G_M^p(Q^2) - G_M^n(Q^2).$$

and

$$\boxed{|\mathcal{F}_V(Q^2)|^2 = \frac{[\mathcal{G}_E^V(Q^2)]^2 + \tau[\mathcal{G}_M^V(Q^2)]^2}{1+\tau}.}$$

$$\sigma_T^{\text{vector}} \propto \tau|\mathcal{G}_M^V(Q^2)|^2; \qquad \sigma_T^{\text{axial}} \propto (1+\tau)|\mathcal{F}_A(Q^2)|^2,$$

$$\sigma_L^{\text{vector}} \propto (\mathcal{G}_E^V(Q^2))^2; \qquad \sigma_L^{\text{axial}} = 0.$$

Topic 1
Shape of Quasielastic Peak
and
"effective spectral functions"

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M. Eric Christy, Hampton University
February 2014

ψ' superscaling Description of the Shape of Quasielastic Peak

The shape of the Quasielastic Peak for the scattering of electrons from nuclear targets is well described by the **ψ'** superscaling formalism. One universal function for all Q2 and all nuclei (**ψ' includes KF in its definition)**

The **ψ'** superscaling function is extracted from the **longitudinal part** of quasielastic electron scattering data on nuclear targets.

See: PhysRevC.71.015501 (2005) , *"Using electron scattering to predict charge-changing neutrino cross sections in nuclei"* , Amaro, J. E. and Barbaro, M. B. and Caballero, J. A. and Donnelly, T. W. and Molinari, A. and Sick,I.).

Also: P. E. Bosted, V. Mamyan arXiv:1203.2262 (2012)

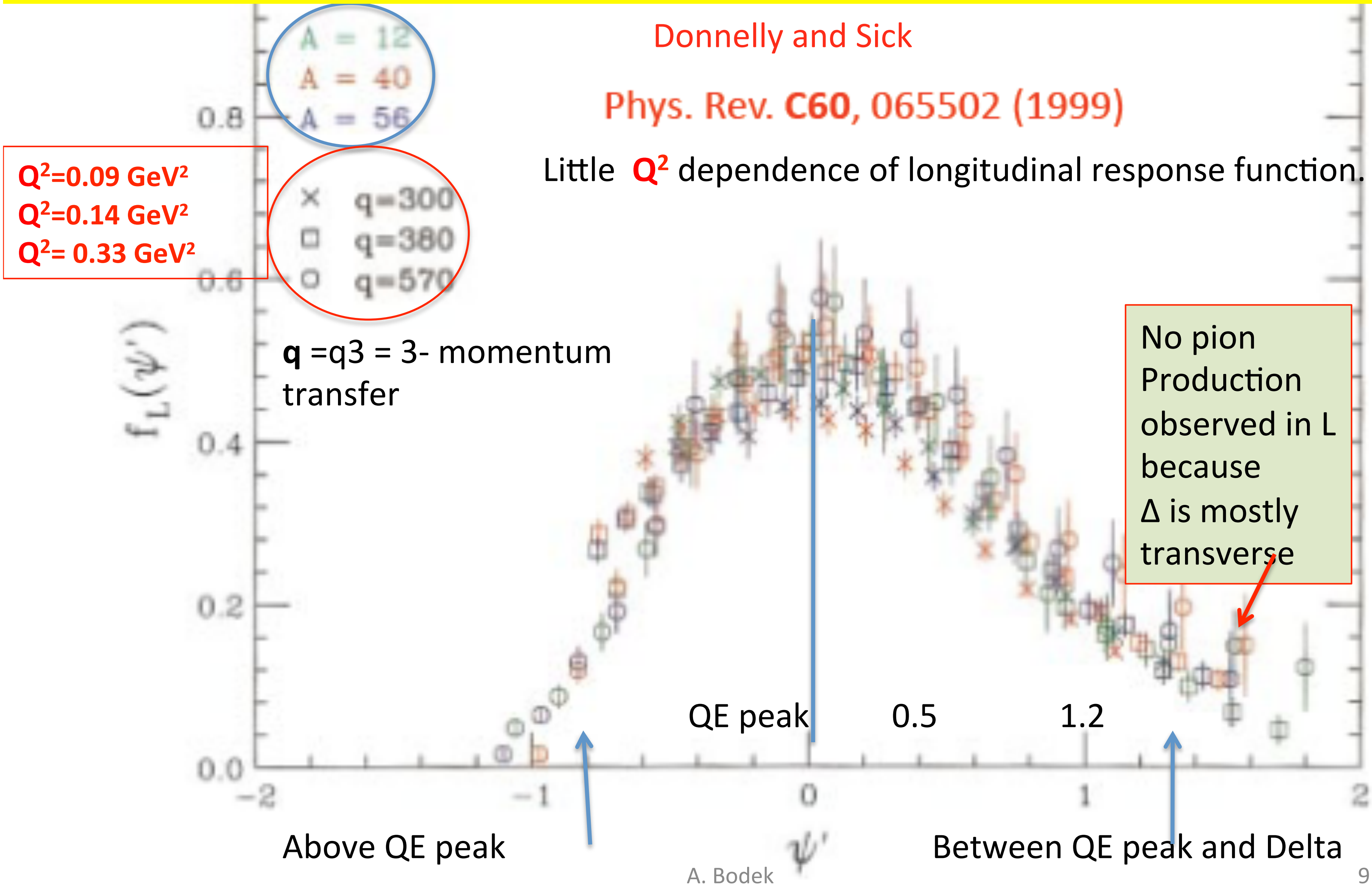
This formalism is also applicable to neutrino scattering from nuclear targets.

The same **ψ'** superscaling function is is used for **longitudinal and transverse and to vector and axial.**

Electron QE scattering: Longitudinal Response Function for

Q^2=0.09 GeV^2 Q^2=0.14 GeV^2 Q^2= 0.33 GeV^2

Longitudinal data data are use to get shape of the universal **ψ’** response function



$$\psi \equiv \frac{1}{\sqrt{\xi_F}} \frac{\lambda - \tau}{\sqrt{(1+\lambda)\tau + \kappa\sqrt{\tau(\tau+1)}}}. \qquad \begin{aligned} \lambda &\equiv \frac{\omega}{2m_N} \\ \boldsymbol{\kappa} &\equiv \frac{\mathbf{q}}{2m_N}. \end{aligned}$$

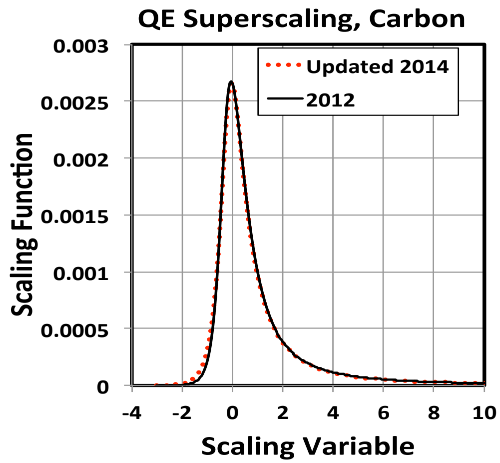
$$\eta_F \equiv \frac{k_F}{m_N} \quad \varepsilon_F \equiv \sqrt{1+\eta_F^2} \quad \xi_F \equiv \varepsilon_F - 1 \quad \tau \equiv |Q^2|/4m_N^2 = \kappa^2 - \lambda^2$$

[1, 2] introducing a dimensionless scaling variable as above,

$$\psi' \equiv \frac{1}{\sqrt{\xi_F}} \frac{\lambda' - \tau'}{\sqrt{(1+\lambda')\tau' + \kappa\sqrt{\tau'(\tau'+1)}}},$$

where $\lambda_{shift} \equiv E_{shift}/2m_N$, $\lambda' \equiv \lambda - \lambda_{shift}$ and $\tau' \equiv \kappa^2 - \lambda'^2$.

Parameters of the superscaling function



A	$k_F(\psi')$ (GeV)	$E_s(\psi')$ (GeV)
2	0.055	0.001
3	0.115	0.001
$3 < A < 8$	0.190	0.017
$7 < A < 17$	0.228	0.0165
$16 < A < 26$	0.230	0.023
$25 < A < 39$	0.236	0.018
$38 < A < 56$	0.241	0.028
$55 < A < 61$	0.241	0.023
$A > 60$	0.245	0.018

Table 1. Values of Fermi-broadening parameter k_F and binding energy parameter E_s used in the ψ'superscaling prediction for different nuclei. The parameters for deuterium (A=2) are to be taken as a crude approximation only.

The 2012 ψ' scaling function is given by:

$$F(\psi') = \frac{1.3429}{k_F}[1 + 1.7119^2(\psi' + 0.19525)^2](1 + e^{-1.69\psi'}) \tag{1}$$

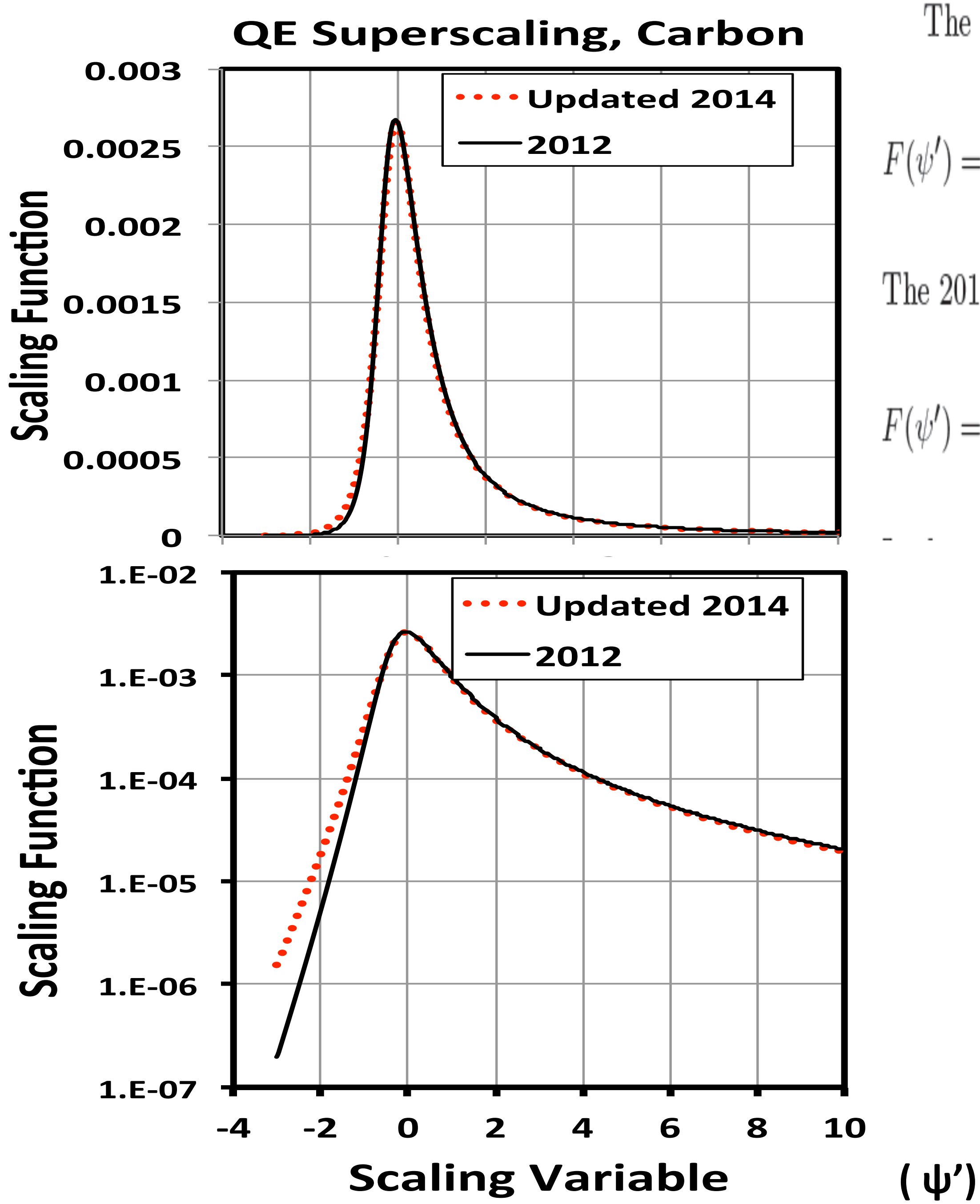
The 2014 ψ' scaling function is given by:

$$F(\psi') = \frac{1.5576}{k_F}[1+1.7720^2(\psi'+0.3014)^2)](1+e^{-2.4291\psi'}) \tag{2}$$

multiplicative Pauli suppression factor given by

$(3/4)(|\vec{q}|/k_F)(1 - (|\vec{q}|/k_F)^2)/12)$

for $|\vec{q}| < 2k_F$, otherwise no correction



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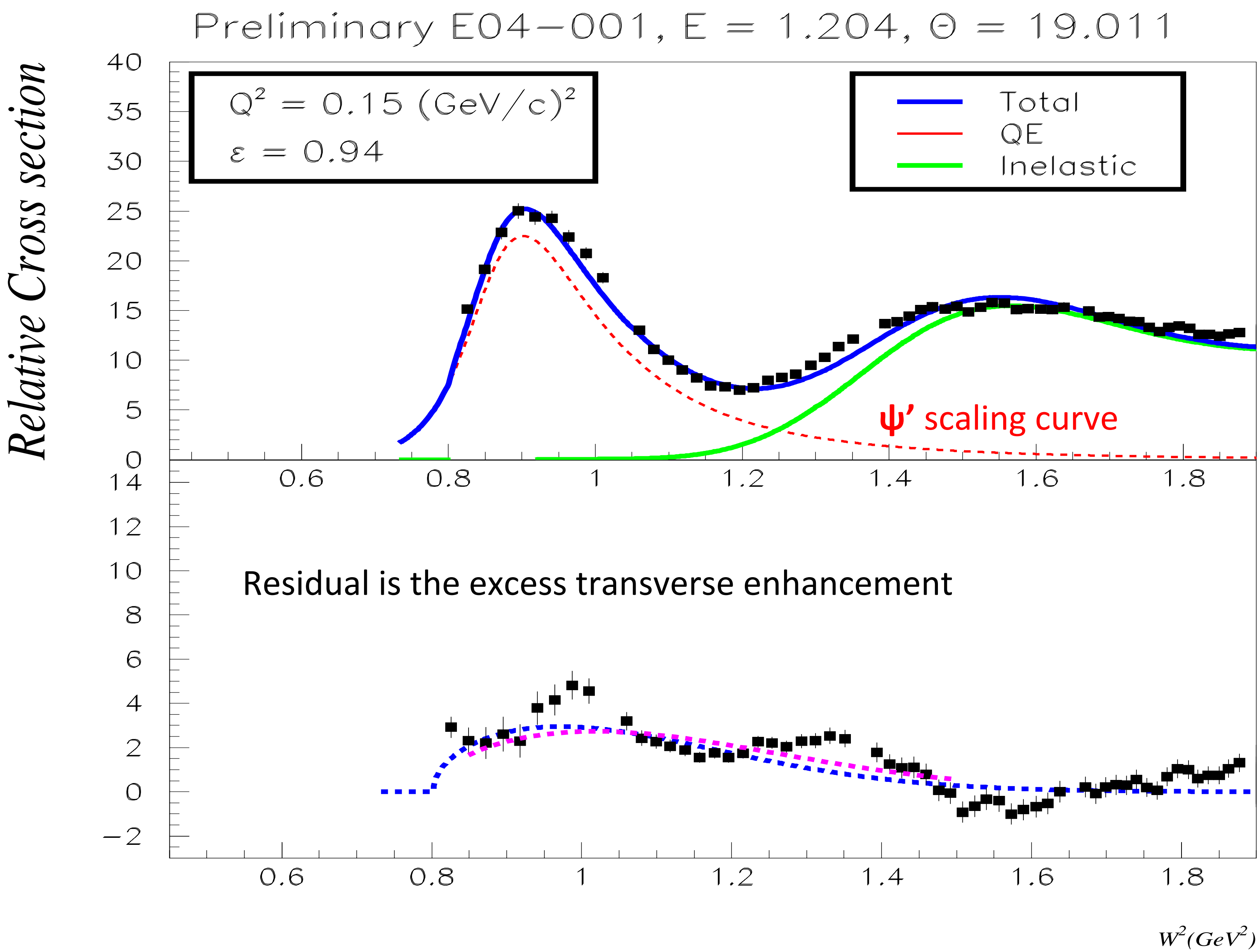

Preliminary E04−001, E = 1.204, Θ = 19.011
Relative Cross section
Q² = 0.15 (GeV/c)²
ε = 0.94
Total
QE
Inelastic
ψ' scaling curve
Residual is the excess transverse enhancement
W²(GeV²)

ψ’ Superscaling Description of the Shape of Quasielastic Peak

The **ψ’** superscaling function for quasielastic scattering from nucleons in a nucleus (in analogy with Bjorken and Nachtman scaling for quarks in the nucleon) accounts for all nuclear effects including initial state momentum and removal energy distribution, two nucleons correlations, and final state interactions. It is derived from experimental electron scattering data.

- **Advantage of ψ’ super scaling formalism**: *The formalism describes the experimentally measured spectrum of the final state lepton, and therefore the correct spectrum of the energy transfer to the nuclear targets nu=E – E*.
- **What is missing from ψ’ super-scaling formalism?** The formalism does not specify in detail how the energy which is transferred to the nuclear target is shared between the final state interacting nucleon and final state spectator nucleus/ nucleons. These details are needed for Monte Carlo simulations of the interaction of the final state nucleus/ nucleons in a neutrino detector, so additional assumptions need to be made.

The superscaling approach is currently used by electron scattering experiments. It should also be used in neutrino MC event generators (but it is not).

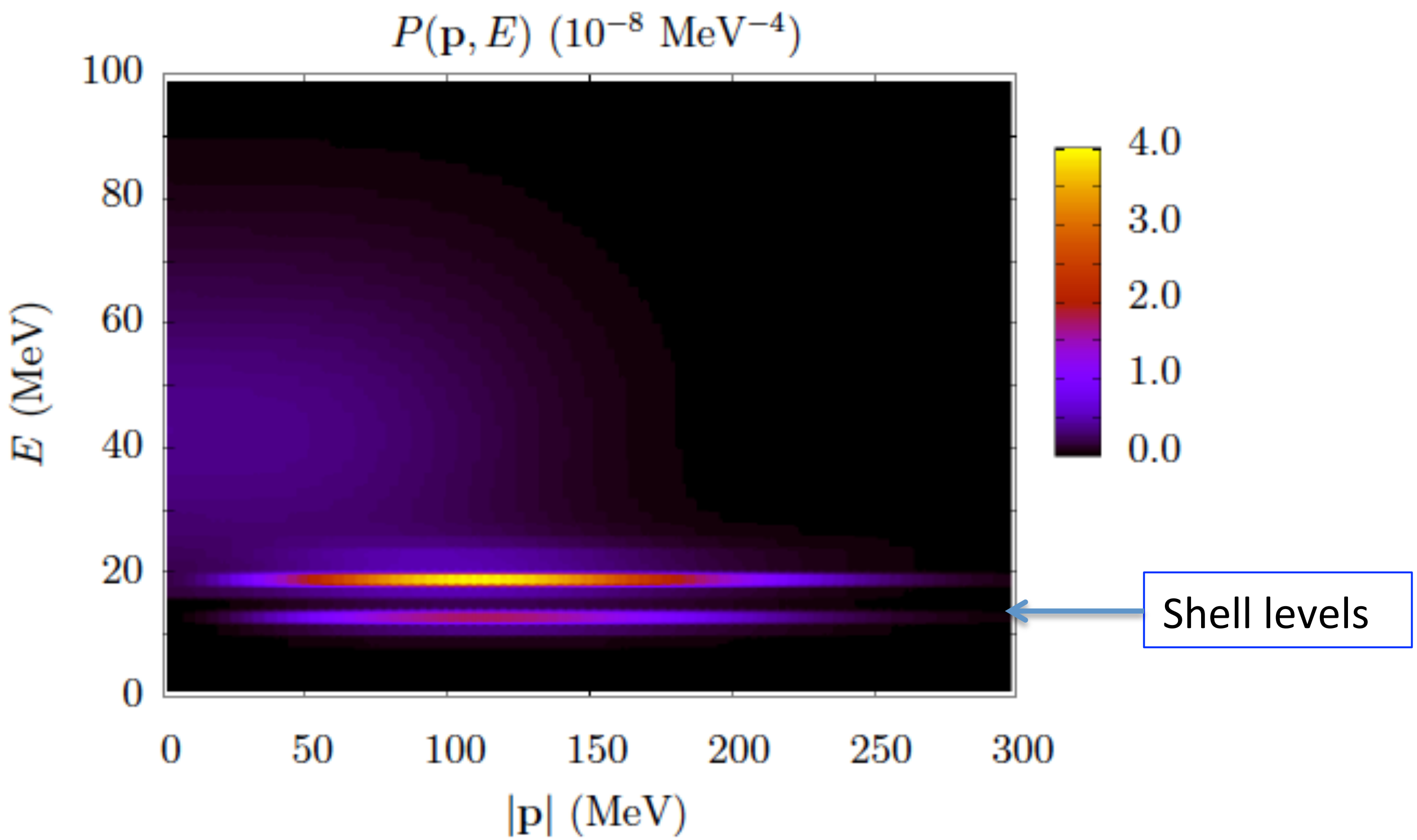


FIGURE 2: Exact spectral function of ${}^{16}_{8}O$ calculated by Omar Benhar.

2D distribution in nucleon momentum and removal energy

Spectral function Description of QE scattering from nuclear targets.

The spectral function description of quasielastic scattering from nuclear targets assumes that the scattering is from bound nucleons with a 2-D distribution in **momentum** and **removal energy**

- **Advantage of the spectral function formalism** is that it is implemented in current neutrino Monte Carlo generators

  where we also have the machinery to implement a model for how the energy transferred to the target is shared between the various nucleons.

- **Disadvantage of the spectral function approach**: If one uses standard spectral functions, then it gives the wrong spectrum.

  It does not account for effect of final state interaction. Therefore, the spectral function prediction for the spectrum of the final state lepton energy (and thus the energy transfer to the nuclear system) is incorrect. Those final state interactions (at the Feynman diagram level) remove strength form the QE peak and move it to the tail of lower energy final state leptons.

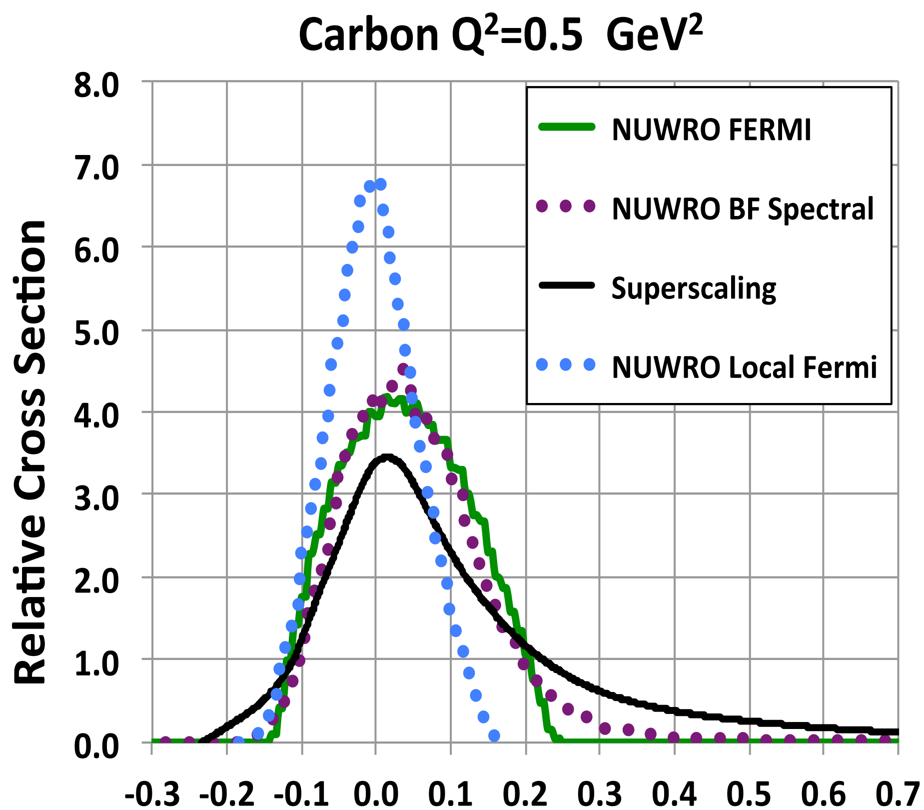
Many in the neutrino community have not yet realized that the spectral function approach gives an incorrect spectrum. This is because most neutrino event generators cannot be easily run in an electron scattering mode, and therefore have not been directly compared to electron scattering data in detail.

Proposed simple solution: Effective Spectral Function

Force the spectral function formalism to get the correct answer for the final state lepton distributions: i.e. Modify the initial state momentum and removal energy distribution such that the predictions are close to the predictions of the superscaling formalism. This effectively includes all the effects of final state interaction in the revised "ef*fective spectral function*"

Specify a reasonable model for the hadronic final state: i.e. specify sharing of the energy of the interacting nucleon and the remaining spectator nucleons.

Comparison of Fermi Gas (local and global) and Benhar Fantoni Spectral Function (as implemented in NUWRO) to the predictions of the ψ' superscaling formalism.



$\Delta\nu = \nu - Q^2/2M_p$ (GeV)

10 GeV Neutrinos.

$\nu = E_\nu - E_\mu$

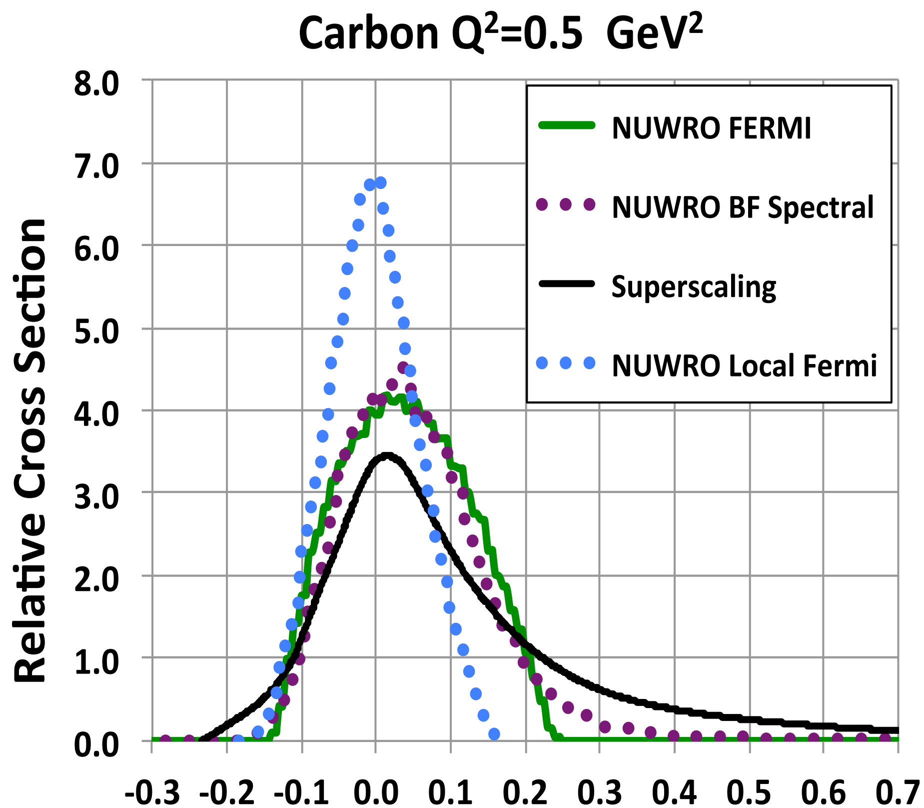
define $\Delta\nu = \nu - Q^2/2M_p$ (GeV)

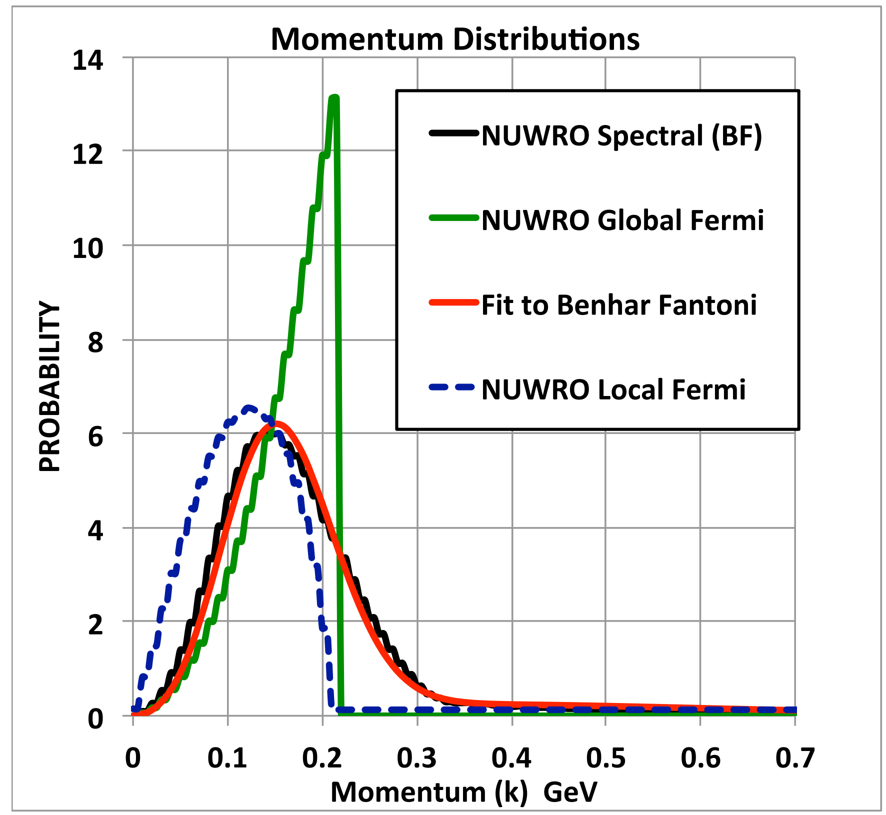
for QE scattering from a free proton $\Delta\nu = 0$

since $\nu = Q^2/2M_p$

Neither Fermi Gas model nor Benhar Fantoni Spectral Function describe the high energy loss tail (large $\Delta\nu$) which originates from final state interaction.

Although the initial momentum distribution and the removal energy are very different in the 1D Fermi Gas and the 2D Benhar-Fantoni spectral Function model, the resulting shape of the QE peak are not very different from each other



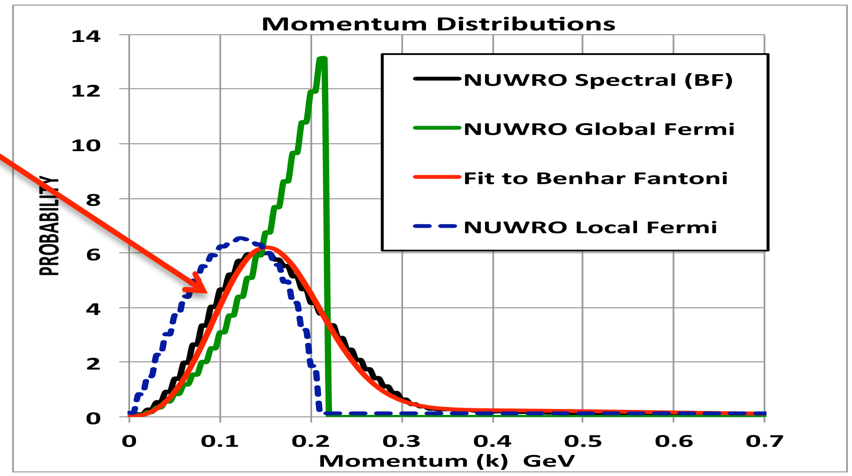


$$\Delta\nu = \nu - \frac{Q^2}{2M_p}$$

Extracting an Effective Spectral Function

We parametrize the momentum distribution using a form of E. Iacopini . This form was used by the NOMAD collaboration in their implementation of an independent nucleon model. We use the same form, but modify the parameters. The probability distribution is defined as.

$$P(k)dk = 4\pi k^2 |\phi(k)|^2 dk.$$



$P(k)$ is parametrized by the following function

$$y = \frac{k}{c_0}$$

$$a_s = c_1 e^{-(b_s y)^2}$$

$$a_p = c_2 (b_p y)^2) e^{-(b_p y)^2}$$

$$a_t = c_3 (y - \beta) e^{-\alpha(y-2)}$$

$$P(k) = \frac{\pi}{4} \frac{1}{c_0 N} [(a_s + a_p + a_t) y^2 \quad ($$

Here, $c_0 = 0.197$, k is in GeV/c, N is a normalization fact to normalize the integral of the momentum distributi from k=0 to k=0.65 GeV to 1.0, and P(k) is in units $(GeV/c)^{-1}$.

Parameter	C12 Benhar-Fantoni
BE (MeV)	2Dspectral
f_{2p2h}	2Dspectral
b_s	1.7
b_p	1.77
α	1.5
β	0.8
c_1	2.823397
c_2	7.225905
c_3	0.00861524
N	0.985

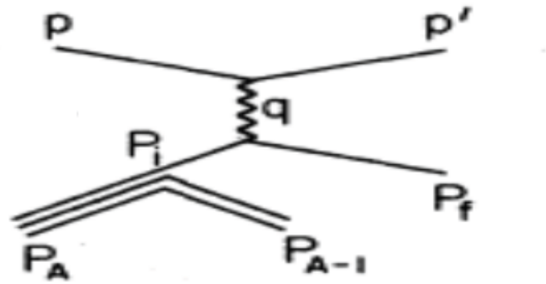
In order to extract an effective spectral function

(a) We vary the 8 parameters of the nucleon momentum distribution

(b) We vary the average effective binding energy (Δ)

(c) Removal energy: In our model the off-shell energy of the spectator nucleon has only two possibilities: *1p1h process and the 2p2h processes*. The relative fractions (f) for these 1p1h and 2p2h processes are assumed to be independent of momentum and allowed to vary in the fit (f_{1p1h}) .

We vary the above 10 parameters till we get a prediction which close to the predictions of the **ψ'** superscaling formalism.



$$P_i = (-\vec{P}_s, M_A - \sqrt{\vec{P}_s^2 + M_{A-1}^{2*}})$$

off shell: $P_i^2 < M_p^2$

on shell: $P_{A-1} = (\vec{P}_s, \sqrt{\vec{P}_s^2 + M_{A-1}^{2*}})$

Fig. 4. 1p1h process: Scattering from an off-shell bound neutron of momentum $\mathbf{P_i} = -\mathbf{k}$ in a nucleus of mass A. The on-shell recoil $(A-1)^*$ (spectator) nucleus has a momentum $\mathbf{P}^*_{\mathbf{A-1}} = \mathbf{P_s} = \mathbf{k}$ and an average excitation energy Δ (effective binding energy). Here $M^*_{A-1} = M_A - M_n + \Delta$. The initial state off-shell neutron has energy $E_n' = M_n - \Delta - \frac{\mathbf{k}^2}{2M^*_{A-1}}$).

1p1h Mean field component.

We assume that this process has a probability of $f_{1p1h} = 1 - f_{2p2h}$ (independent of momentum k)

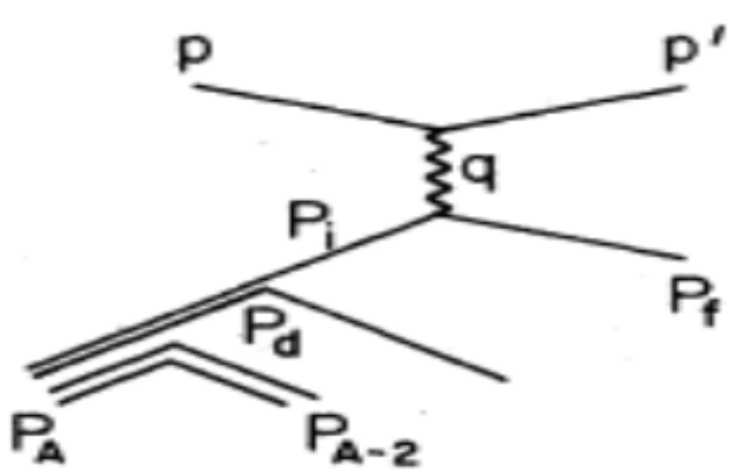
Here the recoil is an (A-1)* excited nucleus

$$\boxed{\begin{aligned} E_n'(1p1h) &= M_A - \sqrt{k^2 + (M^*_{A-1})^2} \\ &= M_n - \Delta - \frac{k^2}{2M^*_{A-1}} \end{aligned}}$$

Removal energy for this process is small since a large nucleus with little energy is balancing the momentum.

$$(M_n')^2 = (E_n')^2 - \mathbf{k}^2$$

2p2h component. Two nucleon corrections (quasideuterons)



$$P_i = (-\vec{P}_s,\ M_d^* - \sqrt{\vec{P}_s^2 + M_p^2}\ )$$

off shell: $P_i^2 < M_p^2$

on shell: $P_{A-2} = (0, M_{A-2}^*)$

on shell: $P_i = (\vec{P}_s, \sqrt{\vec{P}_s^2 + M_p^2}$

$$\boxed{E_n^{'}(2p2h) = M_D - 2(\Delta) - \sqrt{k^2 + M_p^2}}$$

We assume that this process has a probability of f_{2p2h} (independent of momentum k)

Here, the recoil is a single nucleon so the removal energy is large. (momentum k is balanced by a single proton instead of a large nucleus)

$$(M_n^{'})^2 = (E_n^{'})^2 - \mathbf{k^2}$$

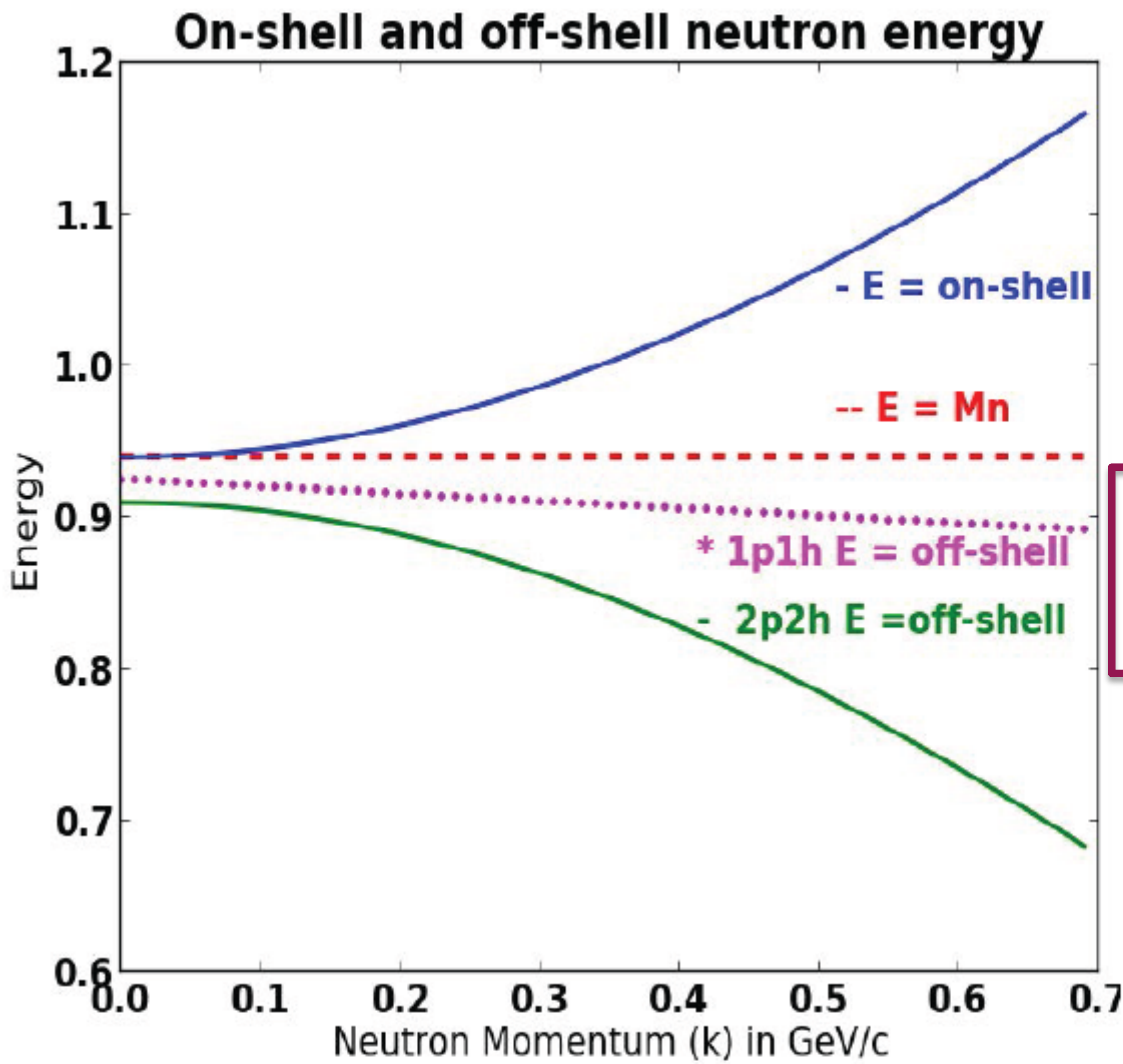


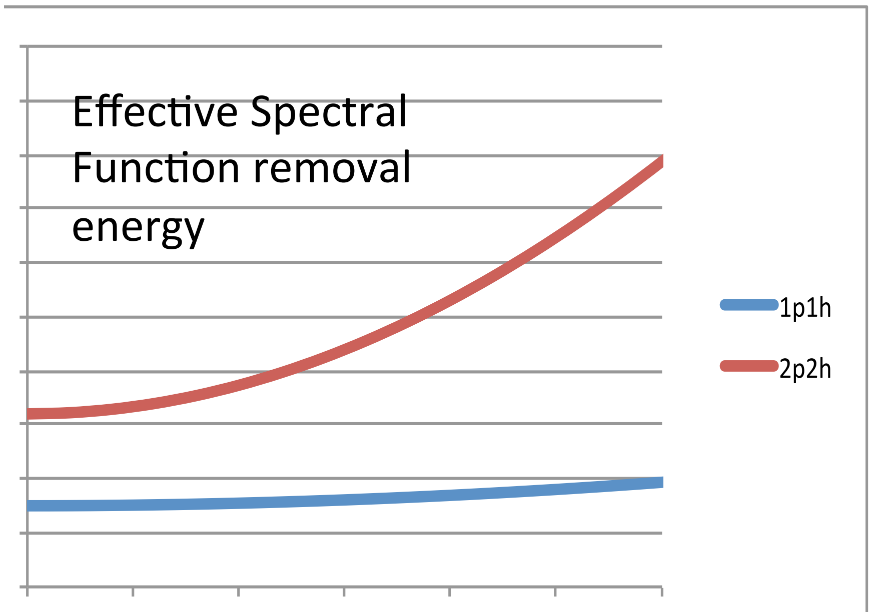
Fig. 3. Comparison of energy for on-shell and off-shell neutrons. The on-shell energy is $E_n = \sqrt{k^2 + M_n^2}$. The off-shell energy is shown for both the 1p1h ($E_n^{'} = M_n - BE - \frac{\mathbf{k^2}}{2M_{A-1}^{*}}$) and 2p2h process ($E_n^{'} = M_D - 2(BE) - \sqrt{k^2 + M_p^2}$)

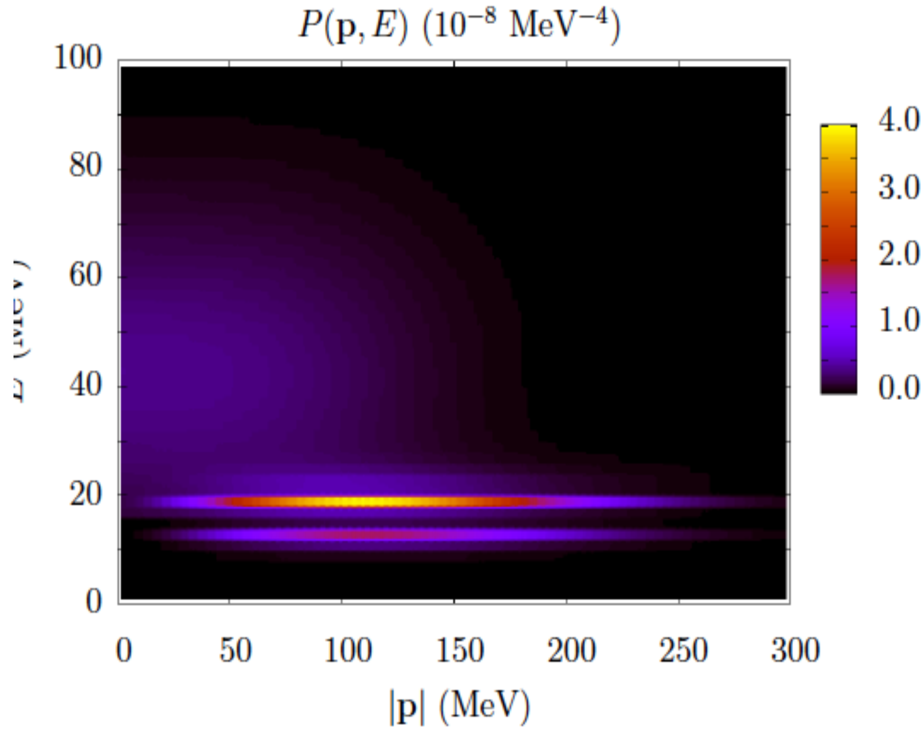
1p1h

$$\boxed{\begin{aligned} E_n^{'}(1p1h) &= M_A - \sqrt{k^2 + (M_{A-1}^{*})^2} \\ &= M_n - \Delta - \frac{k^2}{2M_{A-1}^{*}} \end{aligned}}$$

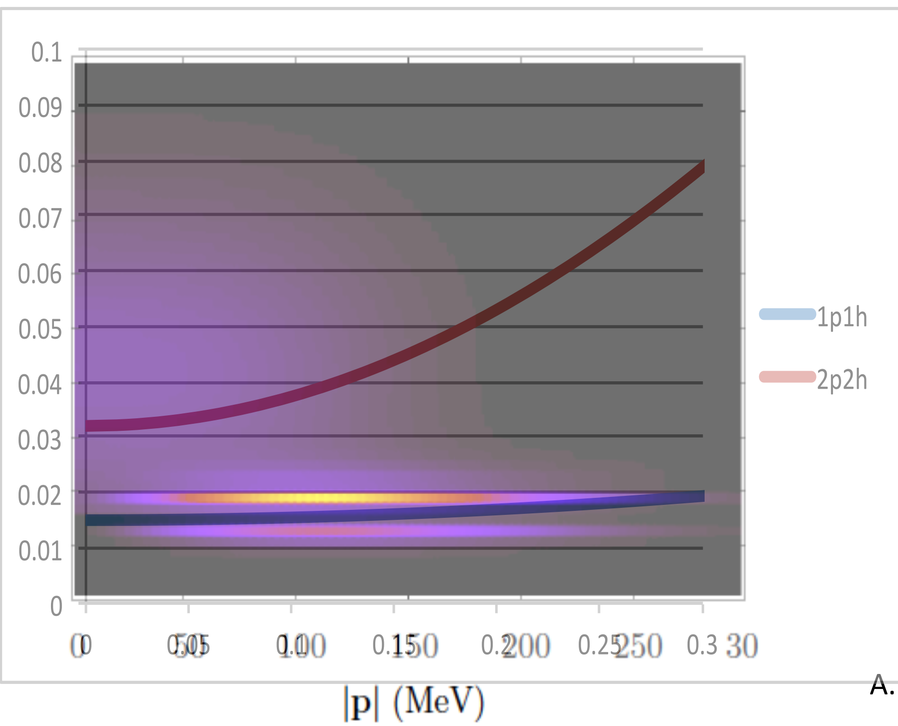
2p2h

$$\boxed{E_n^{'}(2p2h) = M_D - 2(\Delta) - \sqrt{k^2 + M_p^2}}$$





Benhar Fantoi Spectral Function
2D Removal energy vs momtum

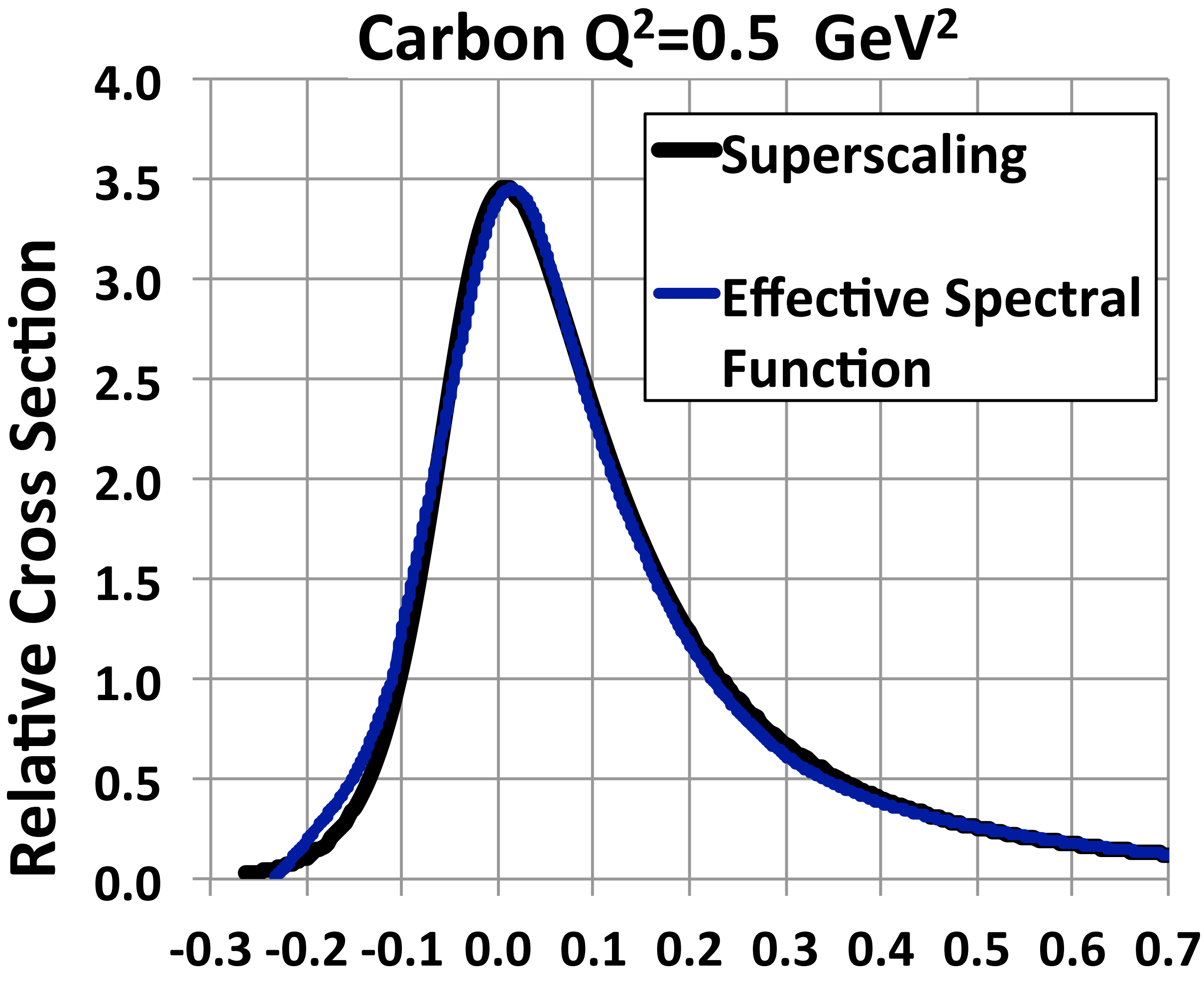


Fitting for the effective spectral function

We compare the predictions of our spectral function the prediction of **ψ'** superscaling.

We vary the 8 parameters of the momentum distribution, vary the binding energy, and vary the fraction of the 2p2h processes.

We only use momenta k<0.65 GeV and normalize all momentum distributions to 1.0 for k<0.65 GeV. We set the momentum distributions to zero for K>0.65 GeV.

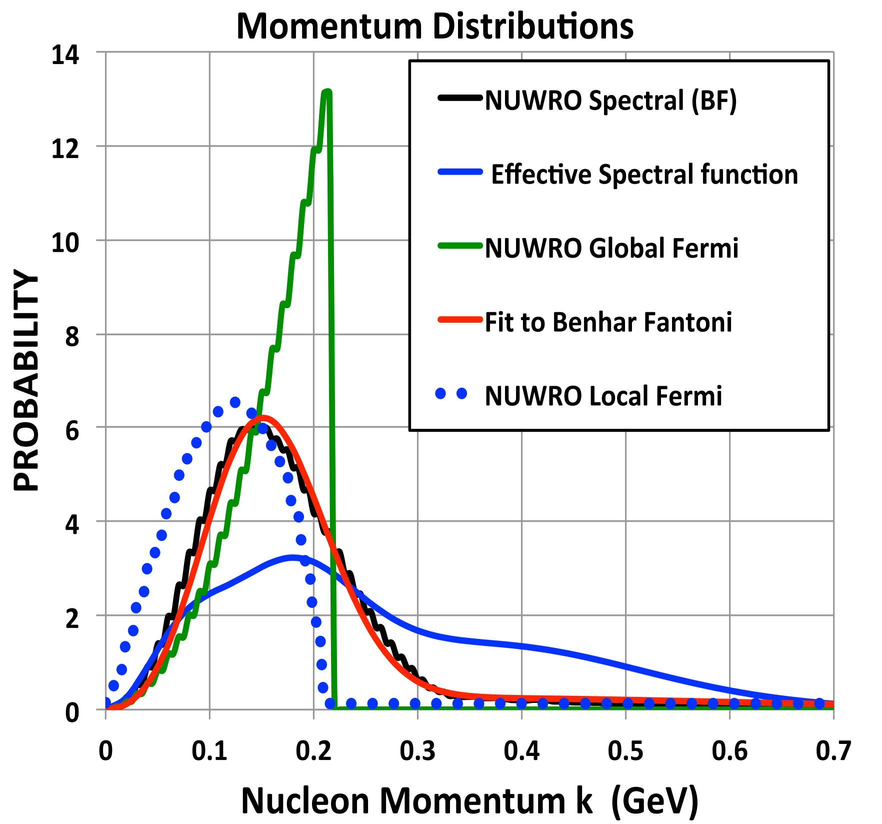


$\Delta\nu = \nu - Q^2/2M_p$ (GeV)

We are able to obtain good agreement with the predictions of super-scaling. Shown is an example at Q^2=0.5 GeV^2

$\Delta\nu = \nu - Q^2/2M_p$ (GeV)

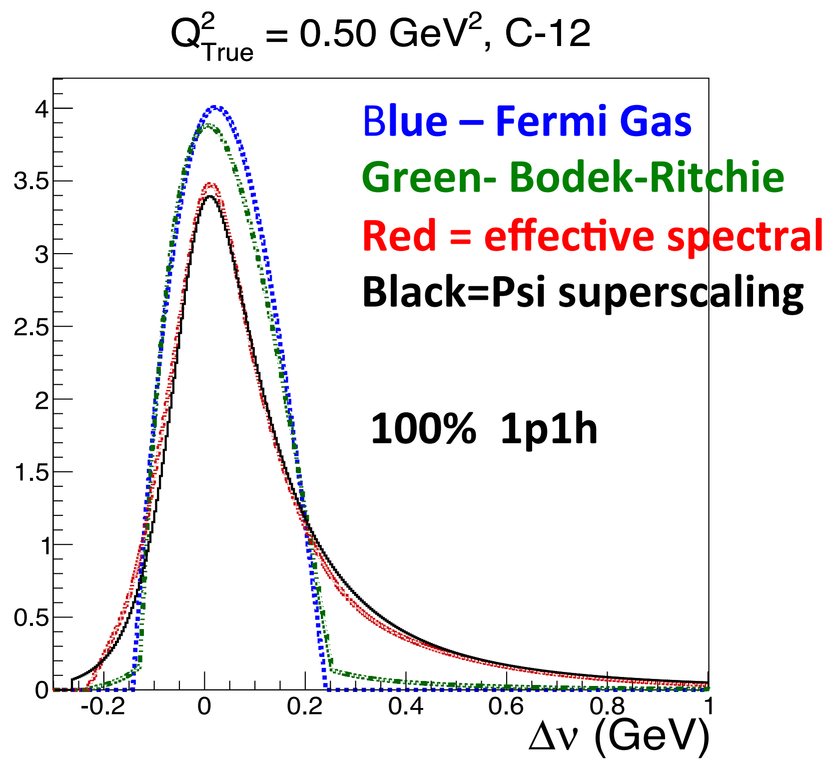
The effective spectral function prediction is as good as that of the Superscaling formalism And therefore preferable to any of the other models.

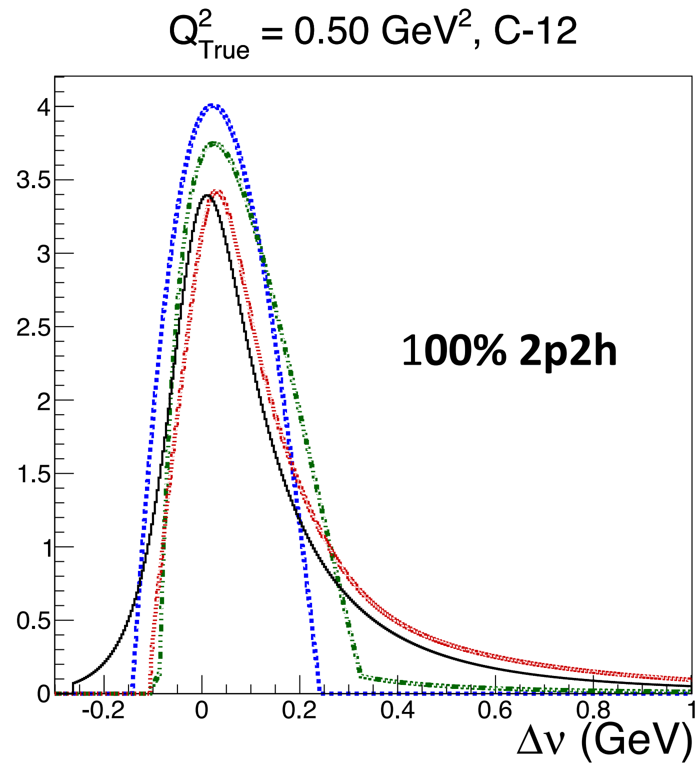


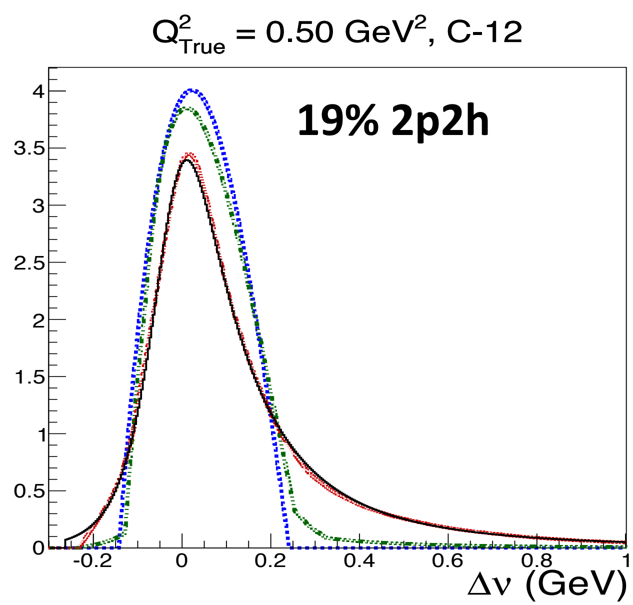
Parameter	C12 Benhar-Fantoni	C12 Effective
Δ (MeV)	2Dspectral	12.5
f_{1p1h}	2Dspectral	0.808
b_s	1.7	2.12
b_p	1.77	0.7366
α	1.5	12.94
β	0.8	10.62
c_1	2.823397	197.0
c_2	7.225905	9.94
c_3	0.00861524	4.36 x10^{-5}
N	0.985	29.64

Table 2. Parameterizations of the Momentum Distribution for Carbon 12 for the Benhar-Fantoni spectral function and for our *"effective spectral function"*. Here, Δ is the effective binding energy, and f_{1p1h} is the fraction of the scattering that occurs via the f_{1p1h} process.

- We increased the fraction of high momentum components to mimic the effect of FSI.
- The fraction of 1p1h is 81%..
- The effective binding energy is 12.5 MeV. Close to the value of the **ψ'** superscaling function.
- We cut of the momentum distribution at K=0.65 GeV and set it to zero for K>0.65 GeV. The normalmization is for K<0.65 GeV.

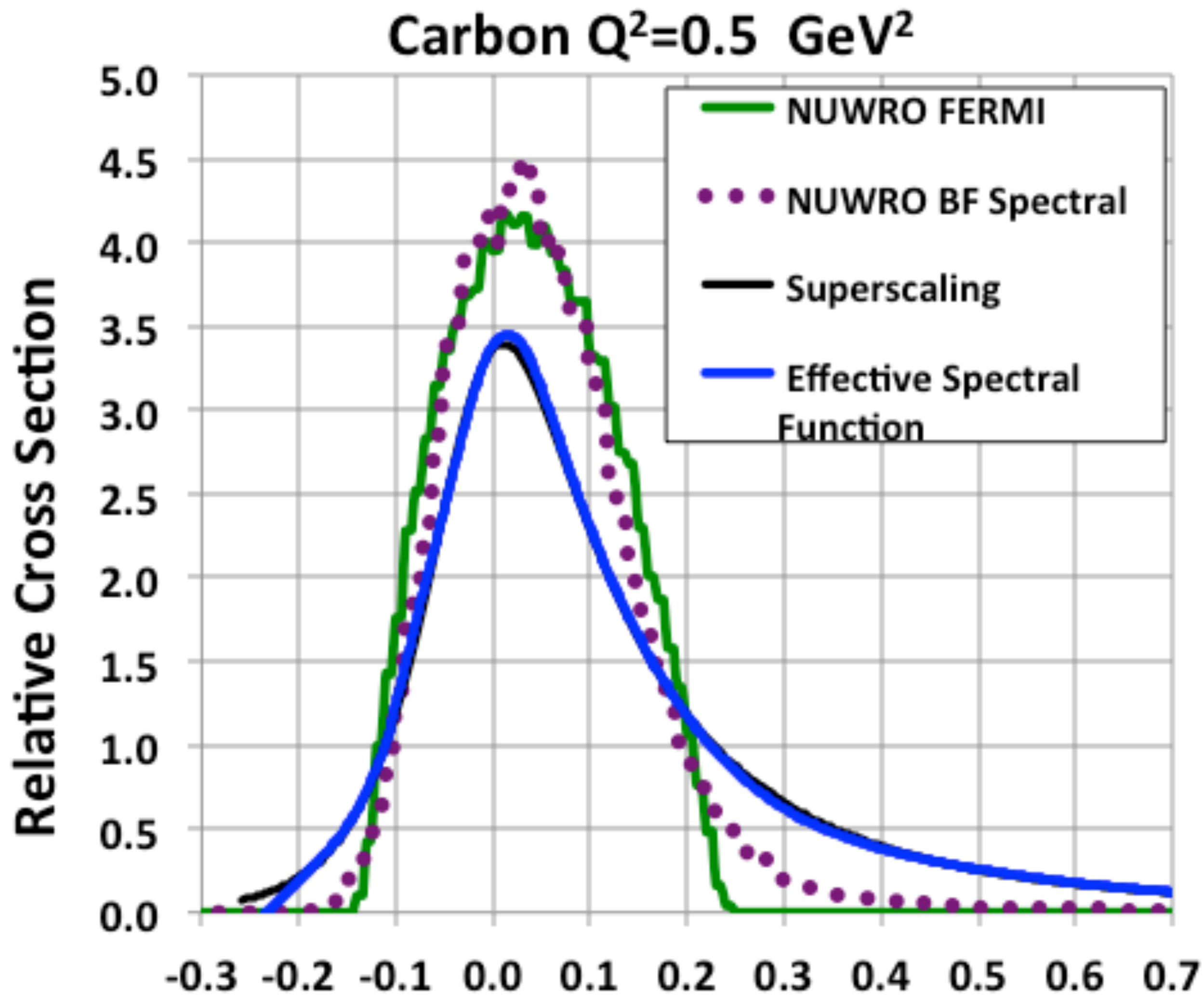






A reasonable effective spectral function (red) could be constructed by increasing the high momentum components, and only including the 1p1h process.

Adding 19% 2p2h makes **the effective spectal function prediction (red)** almost indistinguishable form **superscaling (black)**

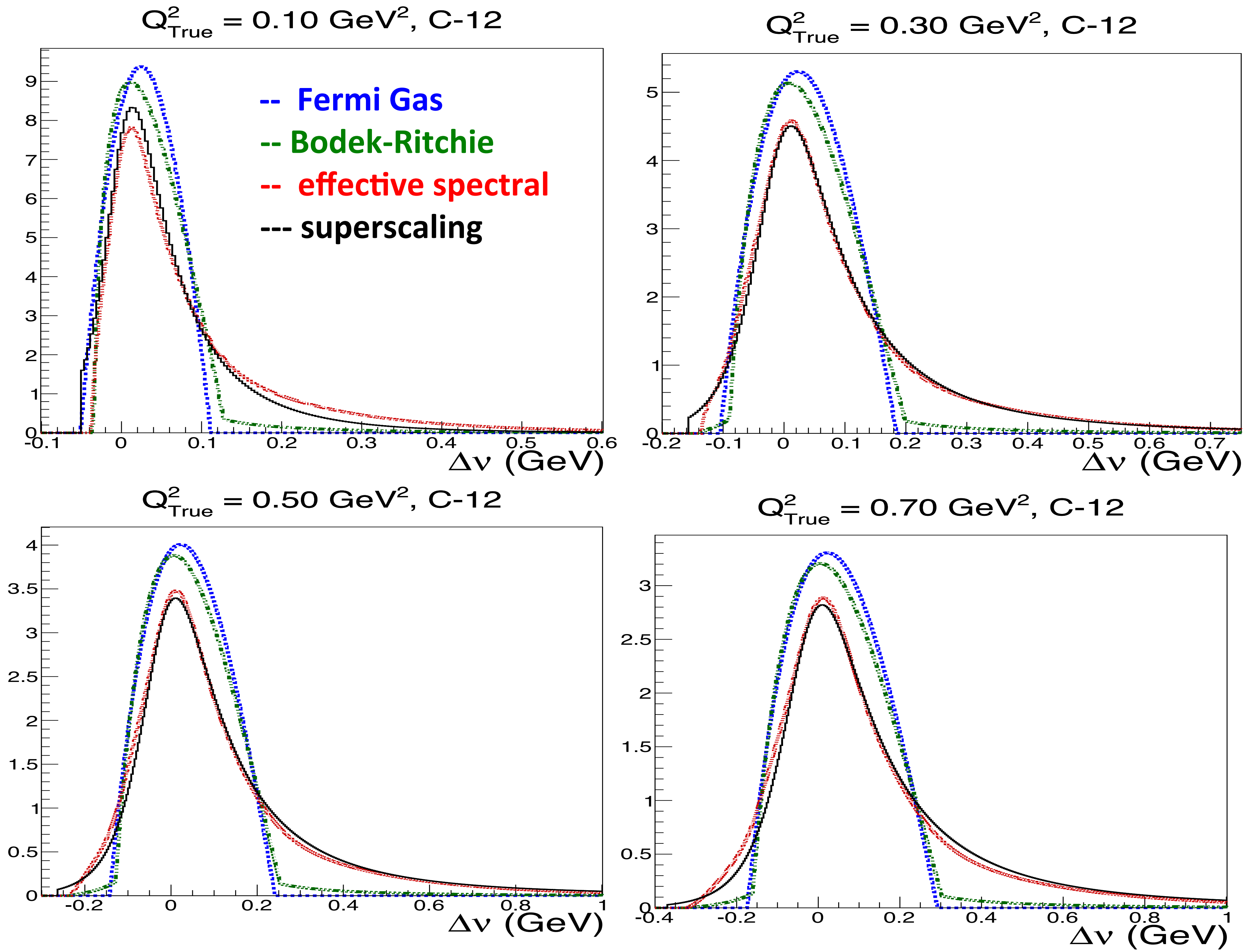


$\Delta\nu = \nu\text{-}Q^2/2M_p$ (GeV)

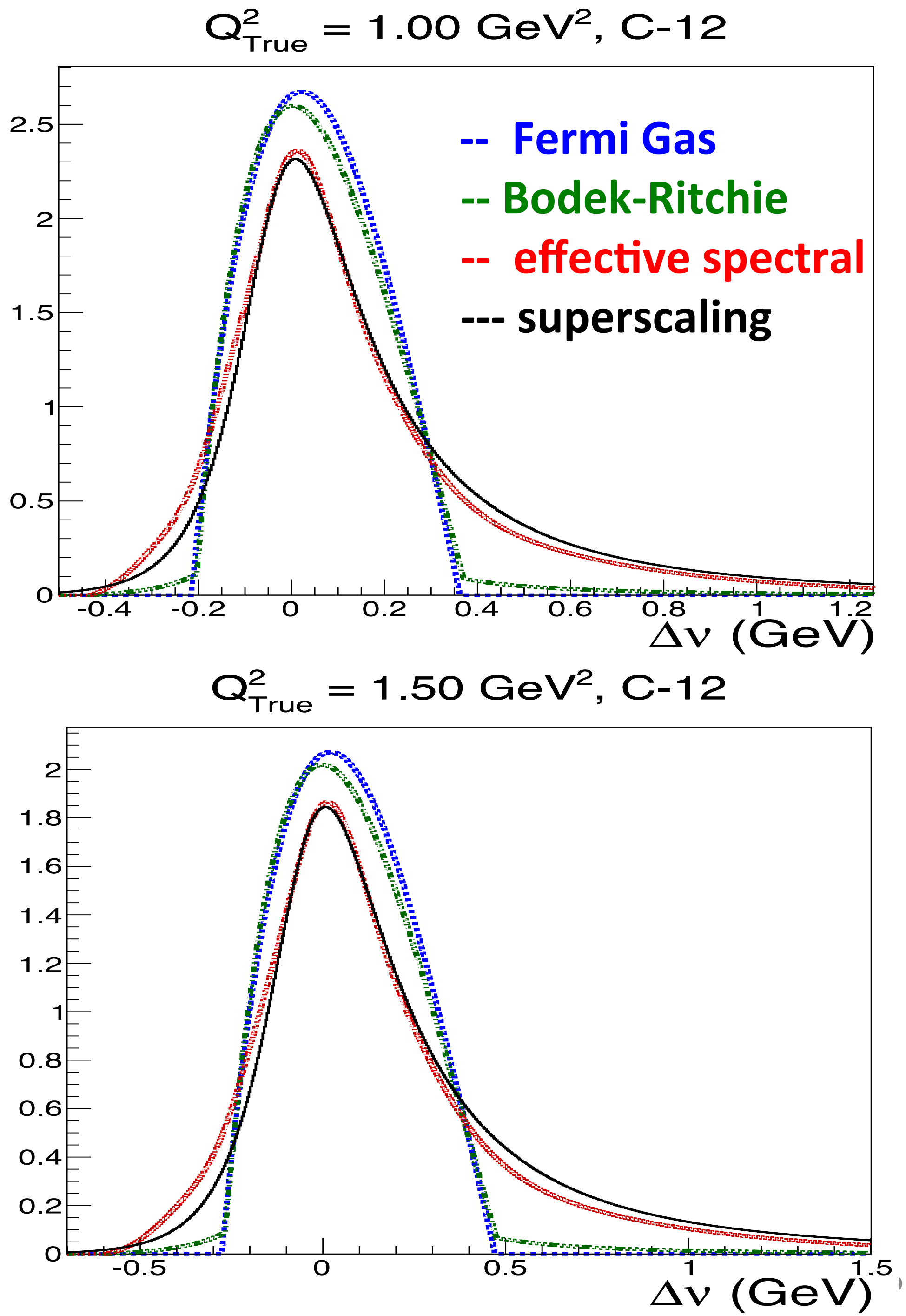
ıappen to QE cross
:tracted from data
Fermi Gas model for the
enhar-Fantoni for the

ted cross section from
ould be too low, unless
ıcluded in the simulation.

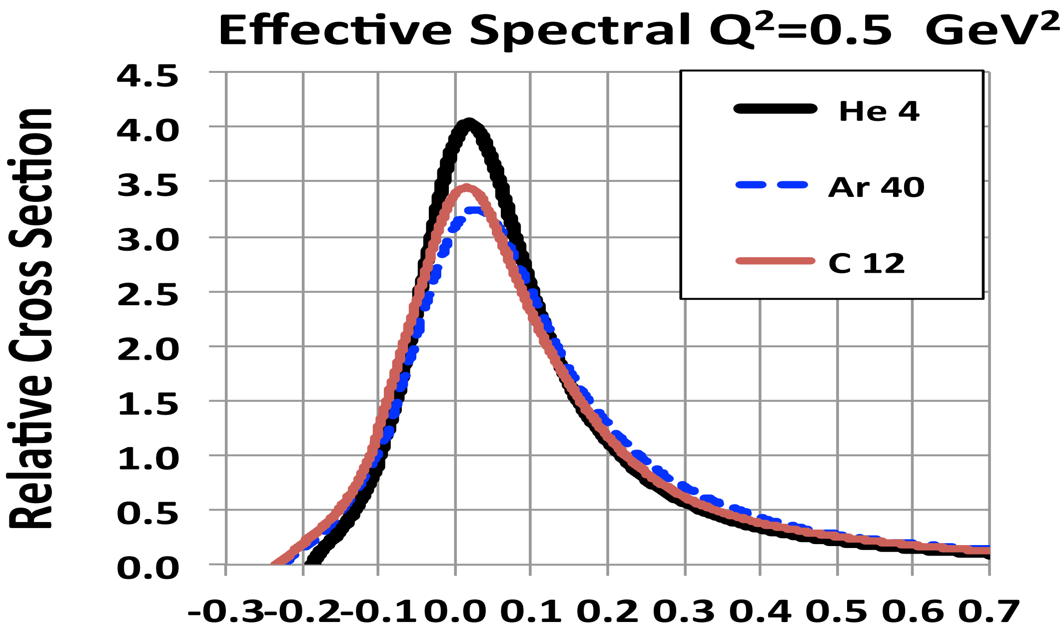
This tail is missing from the Monte Carlo



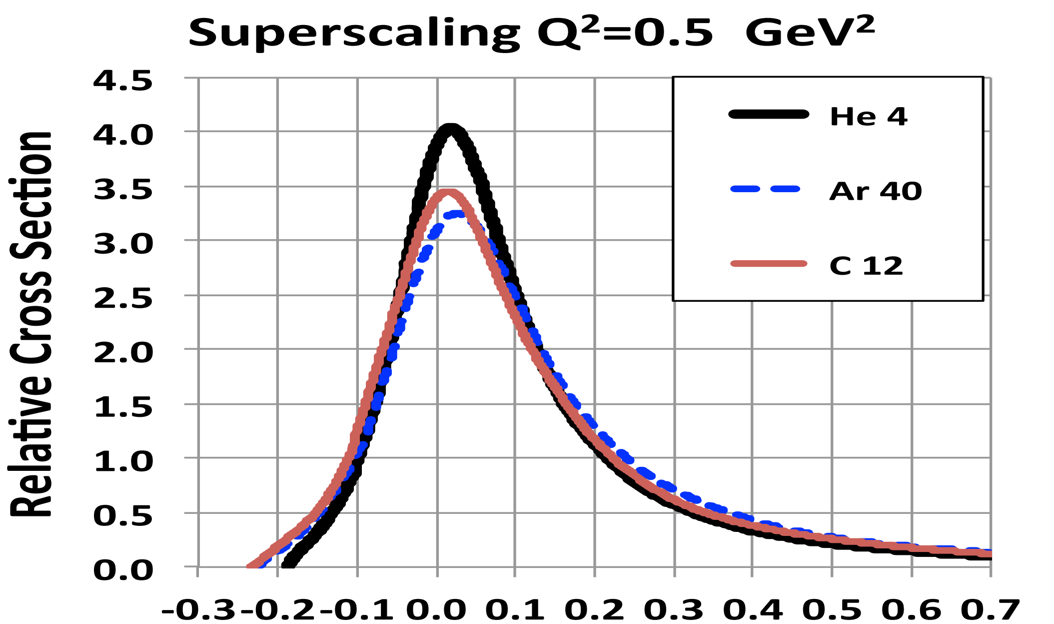
Q^2_{True} = 0.10 GeV^2, C-12
Q^2_{True} = 0.30 GeV^2, C-12
Q^2_{True} = 0.50 GeV^2, C-12
Q^2_{True} = 0.70 GeV^2, C-12
-- Fermi Gas
-- Bodek-Ritchie
-- effective spectral
--- superscaling
$\Delta\nu$ (GeV)



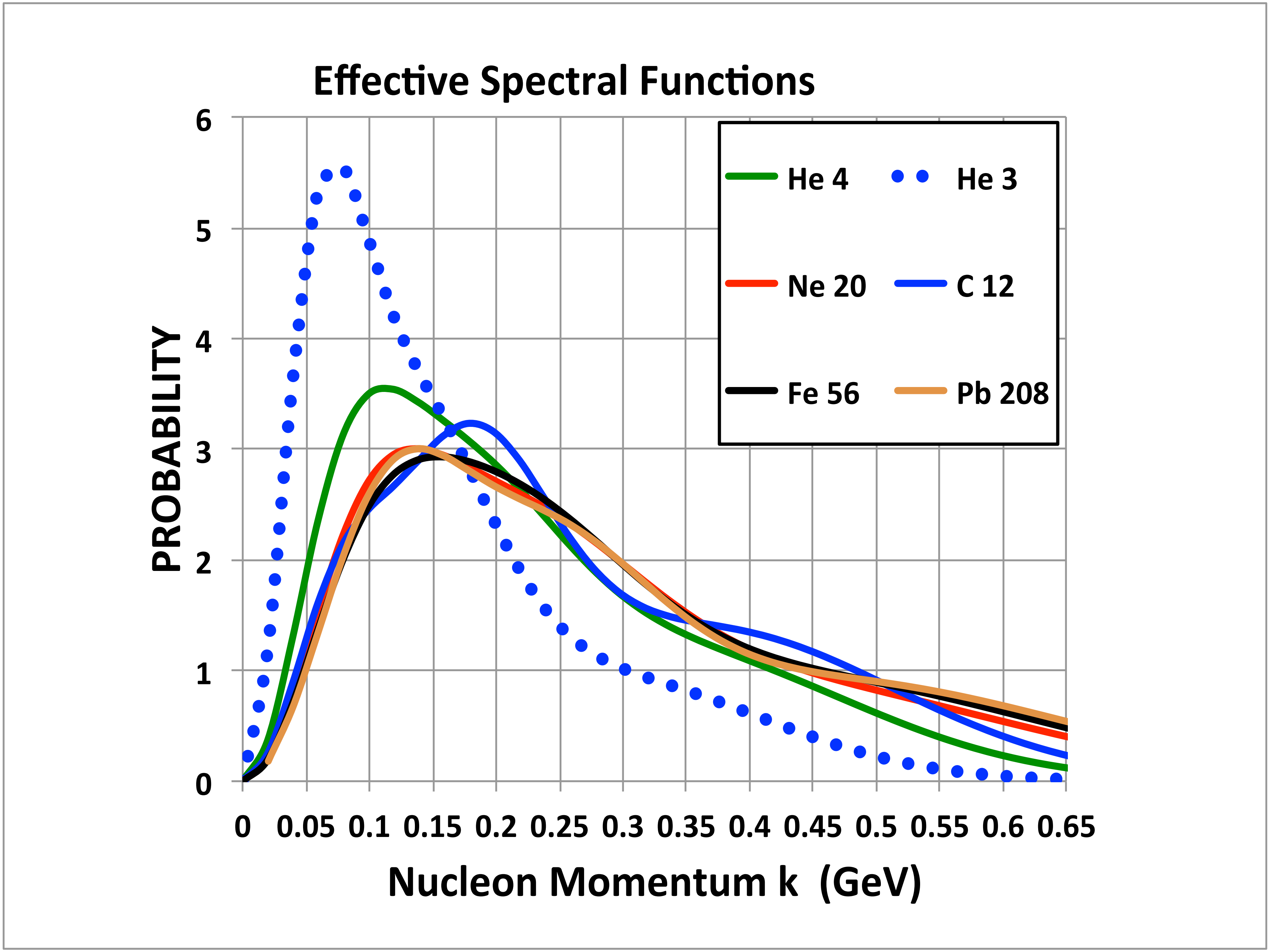
Q^2_{True} = 1.00 GeV^2, C-12
Q^2_{True} = 1.20 GeV^2, C-12
Q^2_{True} = 1.50 GeV^2, C-12
Q^2_{True} = 2.00 GeV^2, C-12
-- Fermi Gas
-- Bodek-Ritchie
-- effective spectral
--- superscaling
$\Delta\nu$ (GeV)



Other nuclei



$\Delta\nu = \nu - Q^2/2M_p$ (GeV)


Effective Spectral Functions
PROBABILITY
Nucleon Momentum k (GeV)
He 4
He 3
Ne 20
C 12
Fe 56
Pb 208
0
1
2
3
4
5
6
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65

Parameter	He-3	He-4	C-12	Ne-20	Al-27	Ar-40	Fe-56	Pb-208
Δ(MeV)	5.3	14.0	12.5	16.6	12.5	20.6	15.1	18.8
f_{1p1h}	0.312	0.791	0.808	0.765	0.774	0.809	0.822	0.896
b_s	3.06	2.14	2.12	1.82	1.73	1.67	1.79	1.52
b_p	0.902	0.775	0.737	0.610	0.621	0.615	0.597	0.585
α	10.93	9.73	12.94	6.81	7.20	8.54	7.10	11.24
β	6.03	7.57	10.62	6.08	6.73	8.62	6.26	13.33
c_1	199.6	183.4	197.0	25.9	21.0	200.0	18.37	174.4
c_2	1.92	5.53	9.94	0.59	0.59	6.25	0.505	5.29
c_3	5.26x10^{-5}	59.0x10^{-5}	4.36 x10^{-5}	221. x10^{-5}	121.5 x10^{-5}	269.0x10^{-5}	140.6 x10^{-5}	9.28x10^{-5}
N	6.1	18.8	29.6	4.53	4.09	40.3	3.66	38.2

Table 3. Parameterizations of the "*effective spectral function*" for various nuclei. Here, Δ is the effective binding energy, and f_{1p1h} is the fraction of the scattering that occurs via the f_{1p1h} process.

Topic 1: “Effective Spectral Functions” Summary

- We have parametrized an “effective spectral function” that provides a much better description of the shape of the distribution of QE events. It can easily be implemented in existing neutrino Monte Carlo event generators.

- This will have an affect on the extraction of neutrino oscillations parameters from neutrino oscillations experiments

- There will be an implementation of this model in NEUT (next month a T2K member is coming to Rochester to work on it).

- We have a preliminary draft of a paper

Topic 2

Transverse Enhancement and Meson Exchange Current Contributions to Quasielastic (QE) Neutrino Scattering on Nuclear Targets

Arie Bodek, Howard Budd

University of Rochester

M. Eric Christy, Thir Narayan Gautam

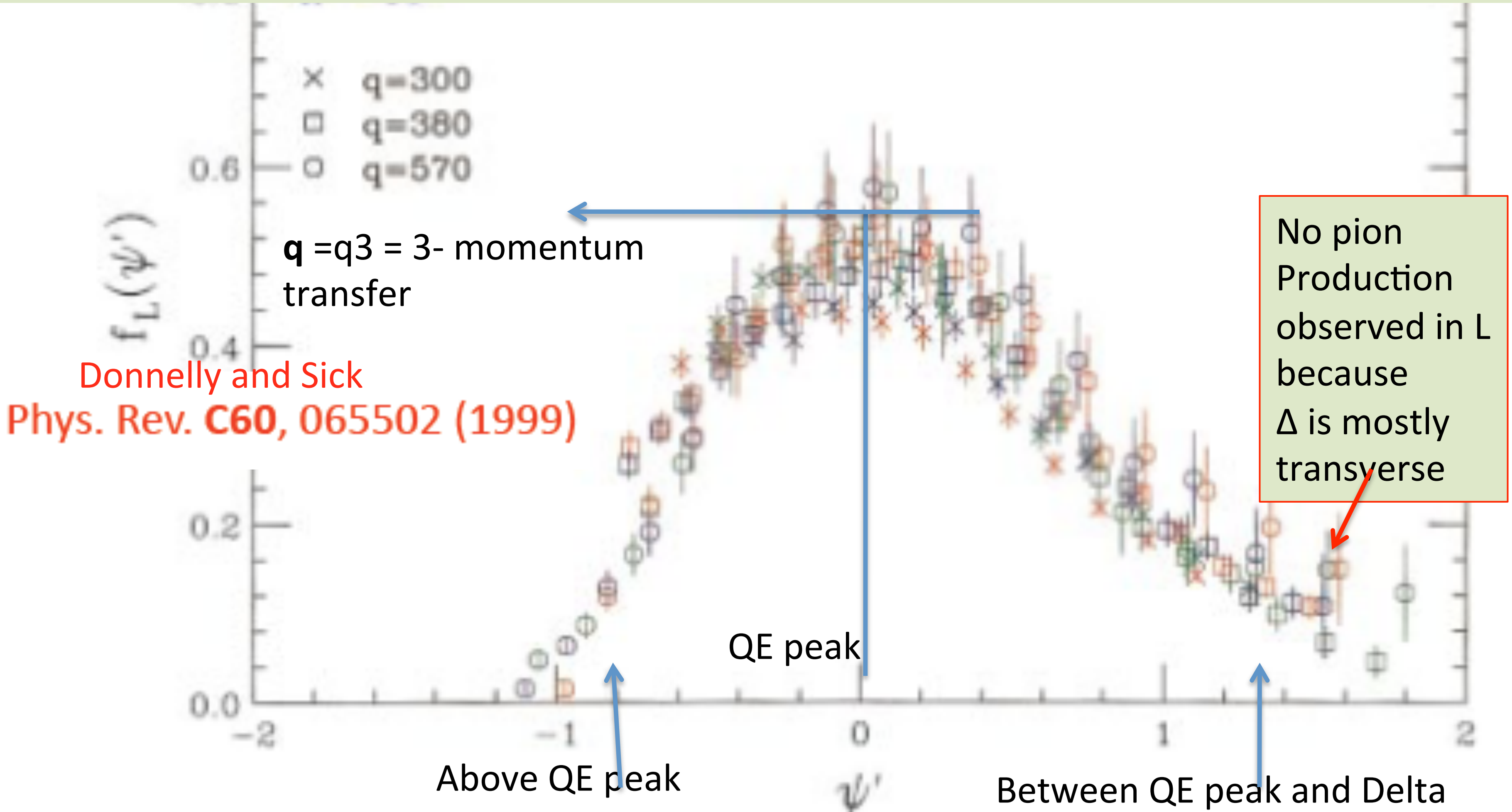
Hampton University

The Superscaling formalism (independent nucleon model) only works For the longitudinal part of the cross sections. These are the data the provides the superscaling function.

IT DOES NOT WORK FOR THE TRANSVERSE PART

Electron QE scattering: Longitudinal Response Function

There are many measurements of differential QE cross section in electron scattering. If we assume free nucleon form factors, and remove their Q2 dependent contribution, **what is left is defined as the nuclear response function** (which is plotted vs the scaling variable Psi)
What is found is that the response function is universal for A>12. It does not depend on momentum transfer, as expected for scattering from independent nucleons

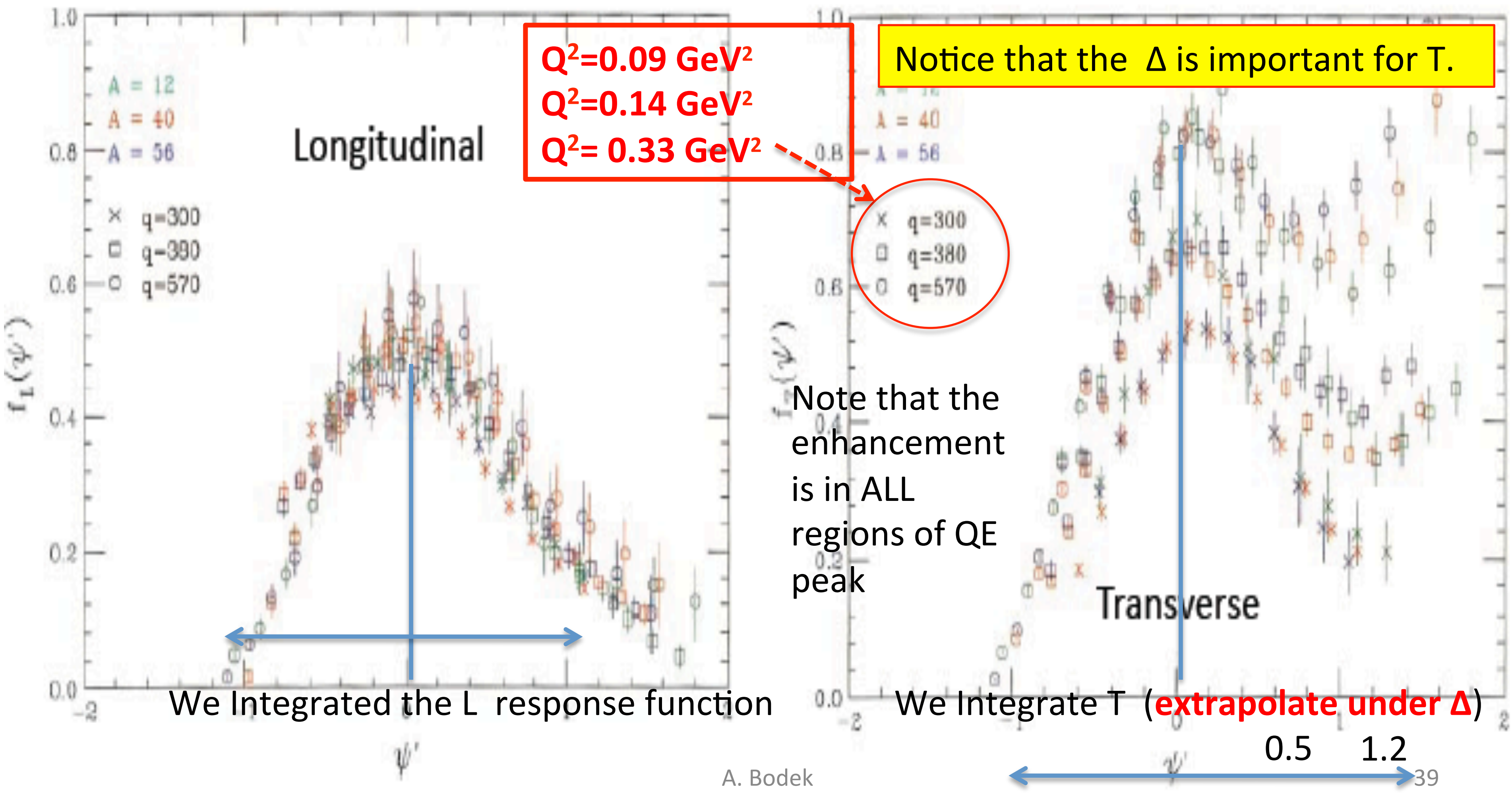


Donnelly and Sick Phys. Rev. **C60**, 065502 (1999)

Response functions (assume free nucleon form factors, and remove their Q2 dependence)

Transverse is enhanced by a Q2 dependent factor R_T

R_T is the ratio of the integrated transverse response function
to the integrated longitudinal response function



Q^2=0.09 GeV^2
Q^2=0.14 GeV^2
Q^2= 0.33 GeV^2

Notice that the Δ is important for T.

Note that the enhancement is in ALL regions of QE peak

We Integrated the L response function

We Integrate T (**extrapolate under Δ**)

0.5 1.2

What is going on with the transverse QE cross section

- The independent nucleon model works very well for the longitudinal part of the QE cross section in electron scattering. For this process, there the dominant 1p1h contribution and also 2p2h contributions from two nucleon correlations (but the interaction is with one independent nucleon only)..

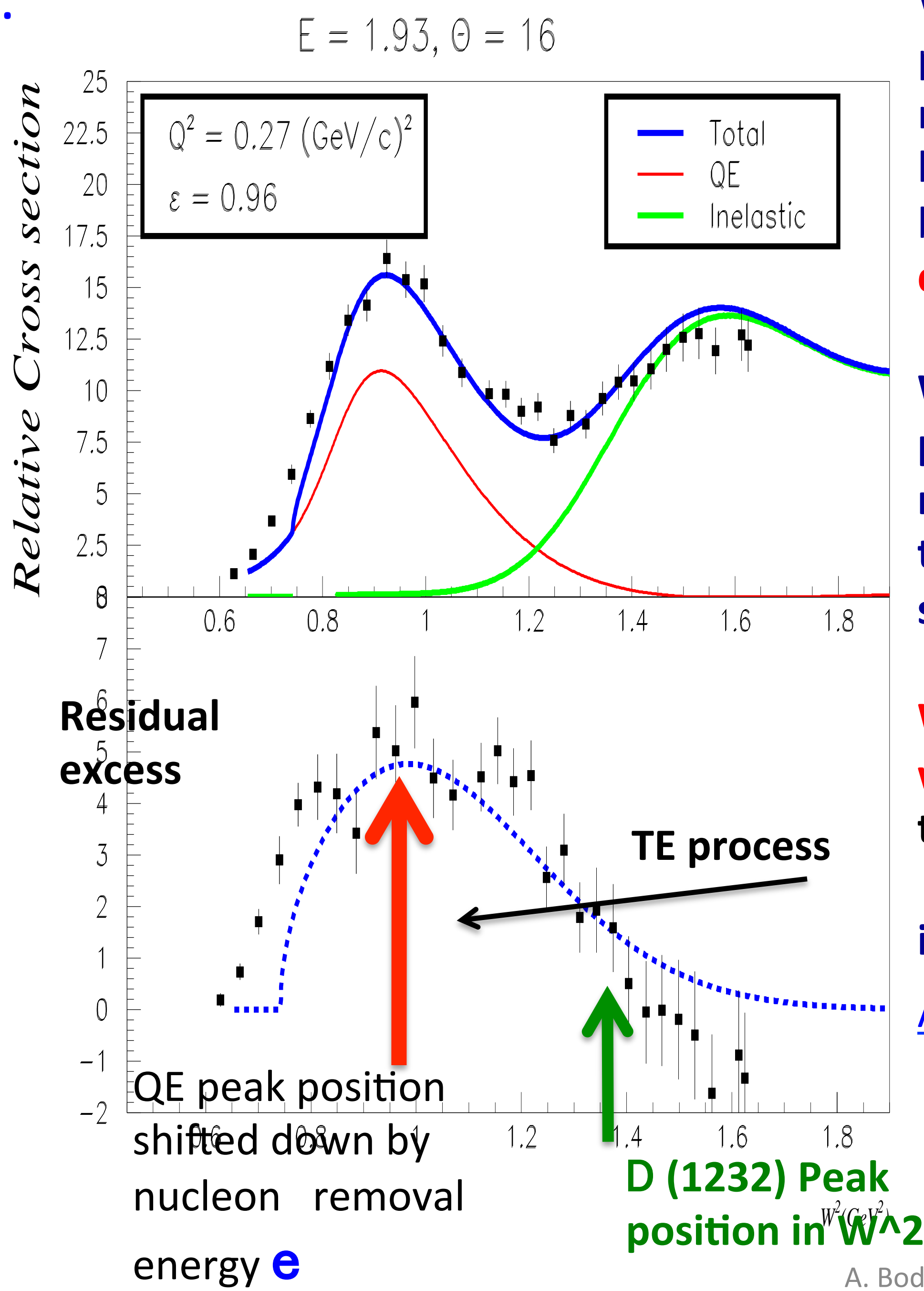
- The data on the transverse cross section indicates that in addition to scattering from independent nucleons, there is an additional source to the cross section when nucleons are bound in nuclei. i.e. two nucleons participate in the scattering process.

- We call this two nucleons source Transverse Enhancement or Meson exchange currents (MEC). Meson exchange currents are a 2p2h process since more that on nucleon is involved. It only happens in the transverse channel.

How do we measure the the enhancement to the Transverse Cross section: We need a reference

- At low Q^2, the longitudinal response is taken as the response function for independent nucleons. For electron scattering, at low Q^2 the longitudinal contribution dominates and can be taken as the reference. Therefore we use the Carlson *[J. Carson et a. Phys. Rev. C 65 024002 (2002)]* results for L/T for **Q^2=0.09 GeV^2 ,Q^2=0.14 GeV^2 and ,Q^2= 0.33 GeV^2**

- At high Q^2, the longitudinal contribution is small, and therefore cannot be a taken as the reference. Instead, we use the predicted QE cross section for the independent nucleon model using Psi' superscaling as the reference.



We compare electron scattering data to the prediction of the sum of an independent QE nucleon model (Psi scaling which is the best known model) plus a Δ resonance smeared by the Fermi gas. → **the sum does not describe the data. There is an excess**

We subtract the sum of QE+ smeared Δ prediction from the data. We integrate the residual excess and divide by the integral of the transverse contribution to the QE cross section and obtain RT (Q2)

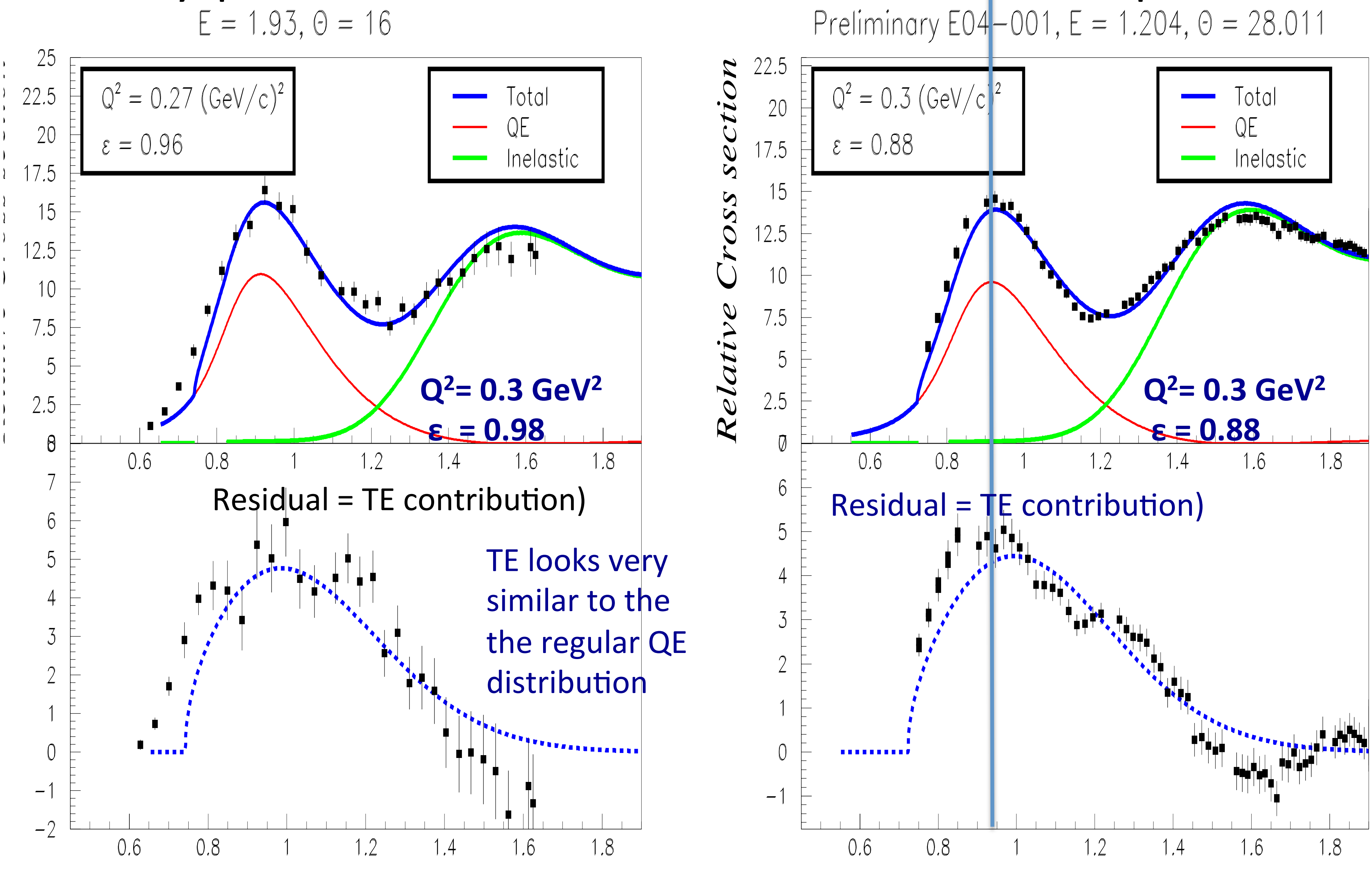
We also extract the peak position and width of the residual excess for the first time.

In an earlier study, we only presented the integral of the TE/MEC excess

[A. Bodek](), [H. S. Budd](), [E. Christy]()

Eur.Phys.J. C71 (2011) 1726 arXiv:1106.0340 [hep-ph]

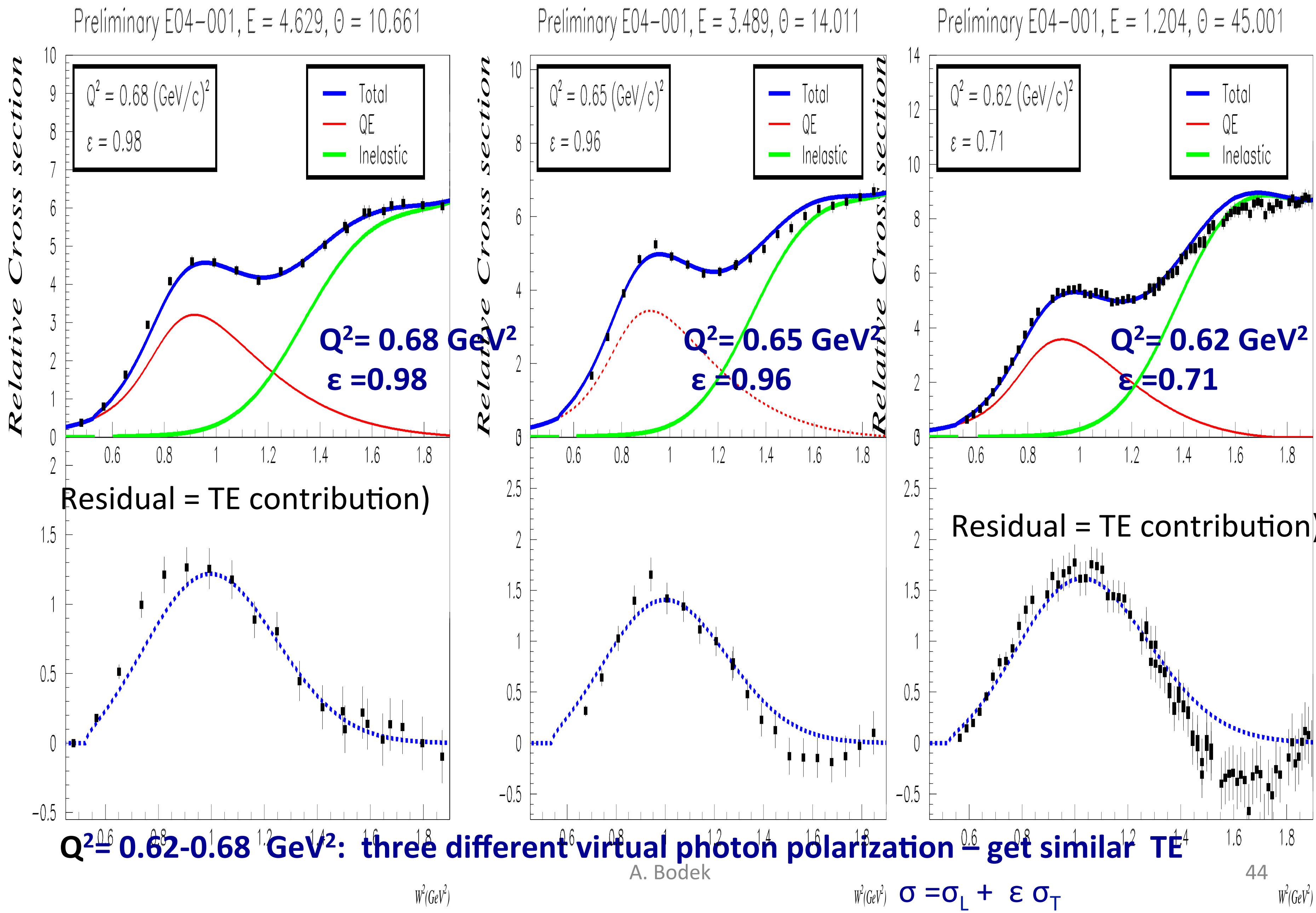
In this study updated the fits to better describe the data. We show a few examples: -



Q^2= 0.3 GeV^2 for two different virtual photon polarization – get same same TE

$\sigma = \sigma_L + \varepsilon\, \sigma_T$

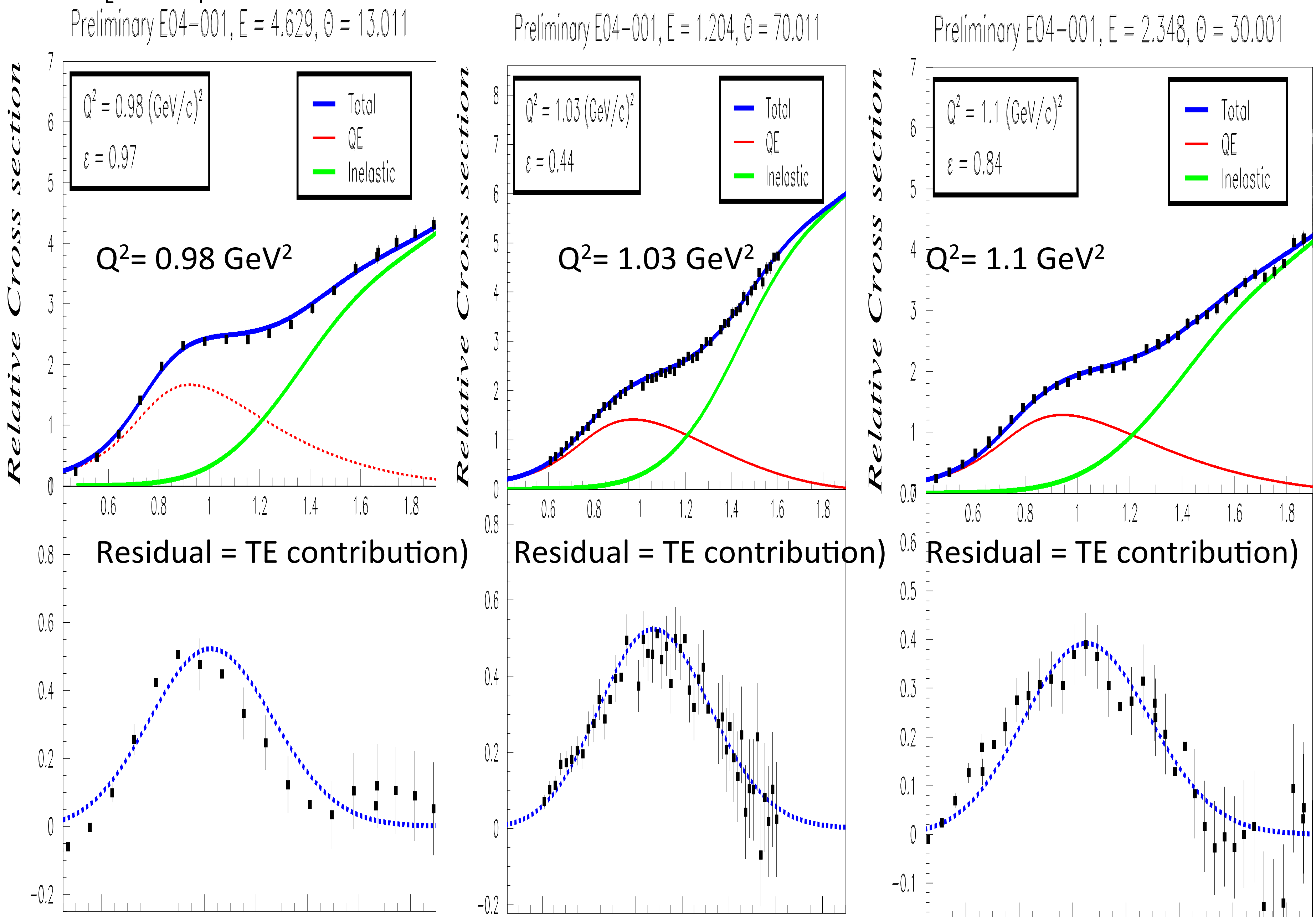
$\sigma = \sigma_L + \varepsilon \sigma_T$



Q^2= 0.62-0.68 GeV^2: three different virtual photon polarization – get similar TE

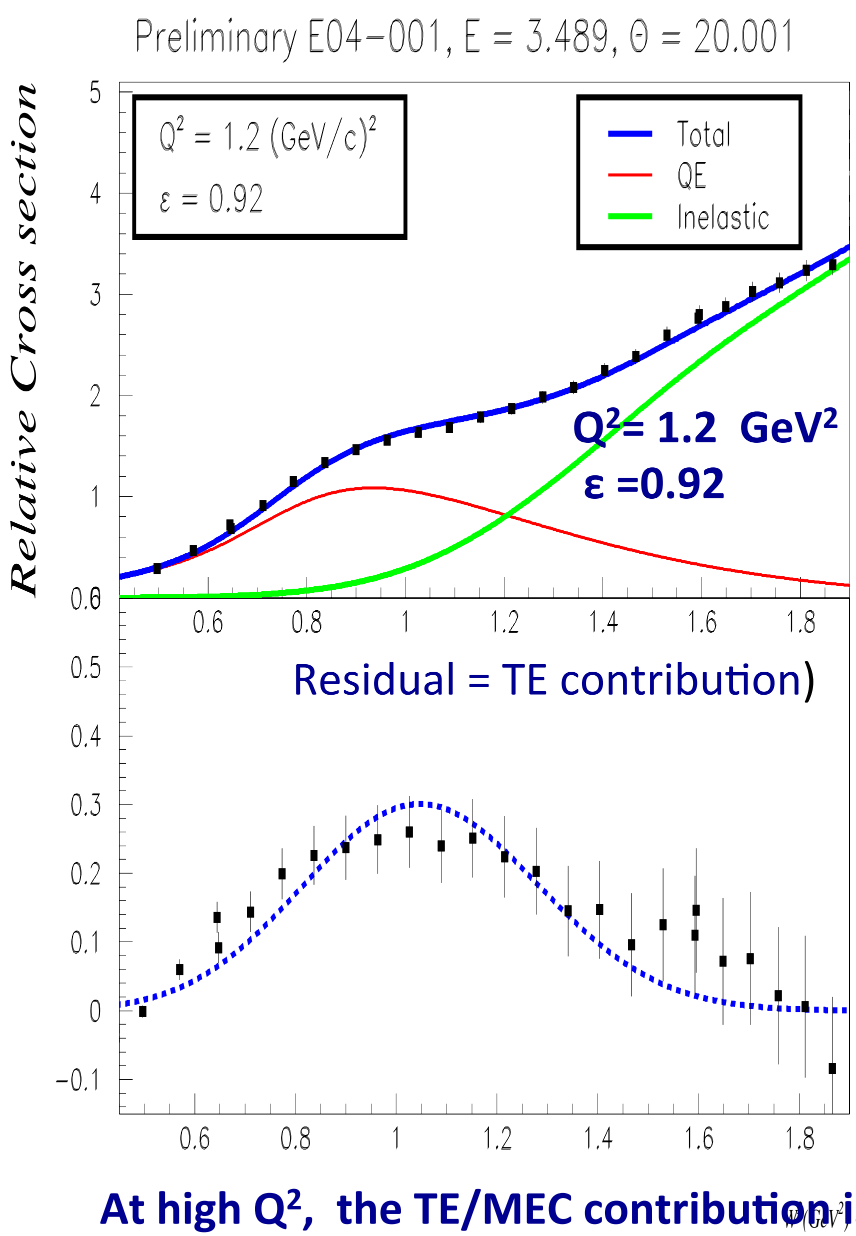
$\sigma = \sigma_L + \varepsilon \sigma_T$

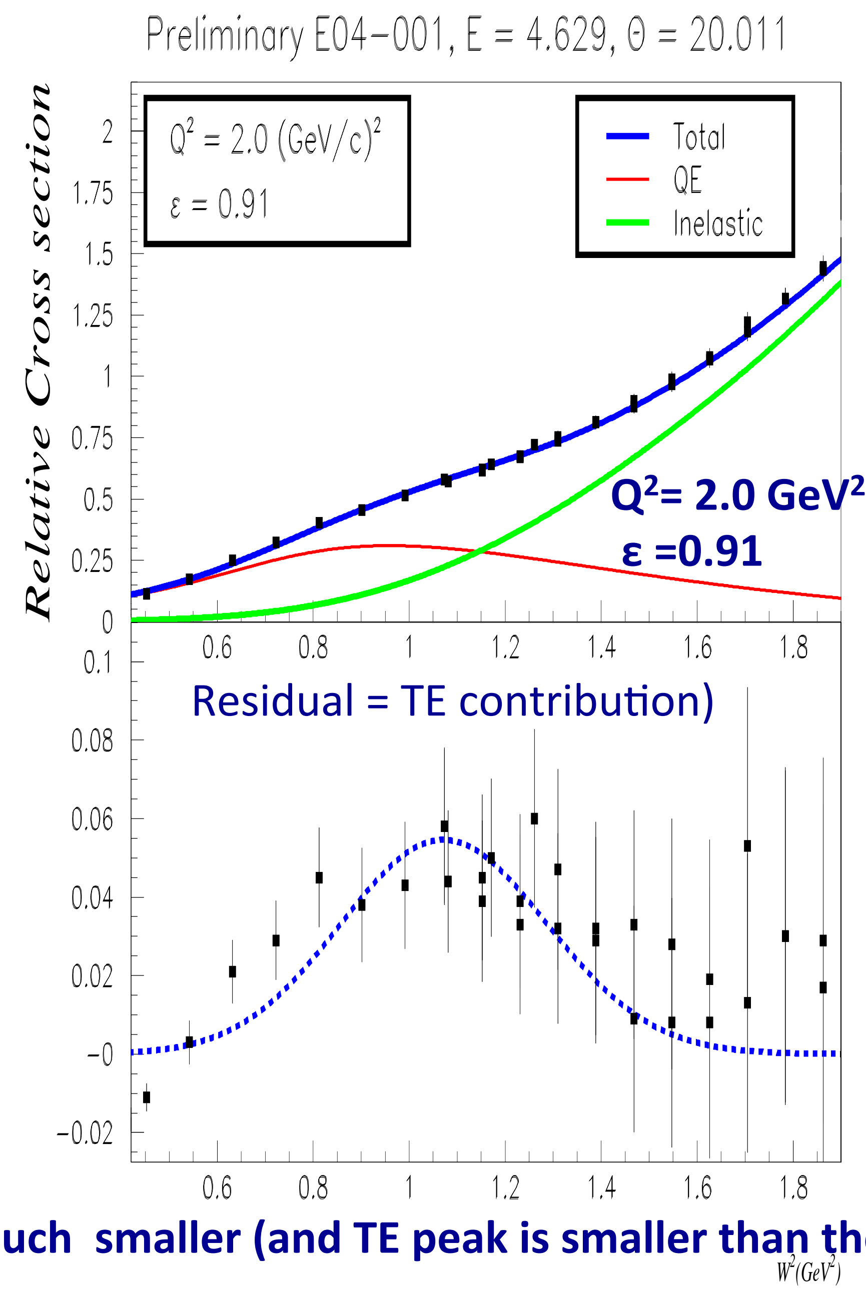
$\sigma = \sigma_L + \varepsilon\, \sigma_T$



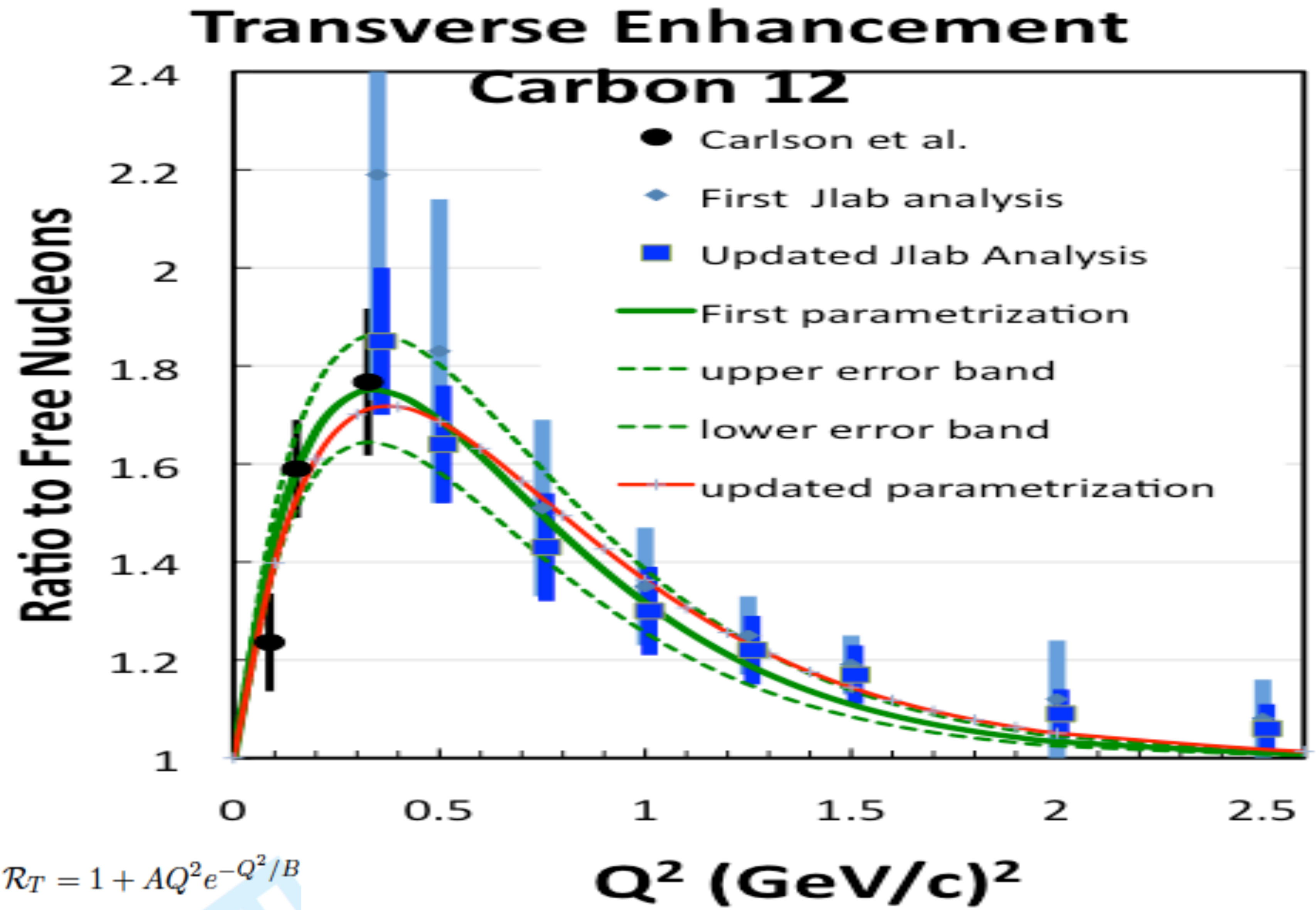
Q^2= 0.98- 1.1 GeV^2: three different virtual photon polarization – get similar TE

$\sigma = \sigma_L + \varepsilon\, \sigma_T$





At high Q^2, the TE/MEC contribution is much smaller (and TE peak is smaller than the QE contribution



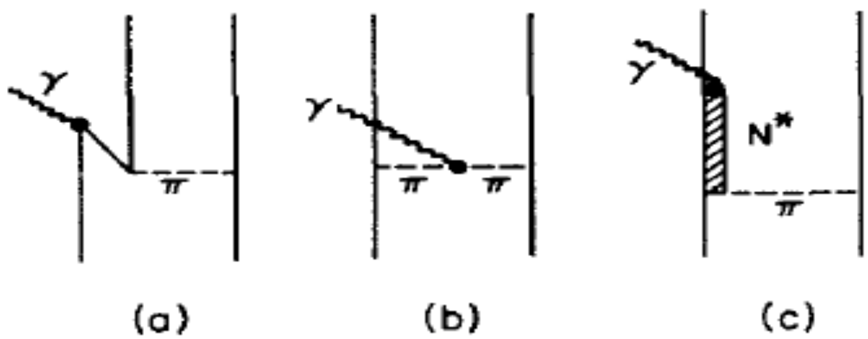
$$\mathcal{R}_T = 1 + AQ^2 e^{-Q^2/B}$$

Updated parameterization A= 5.19 and B= 0.376

The original fit(A=6.0 and B=0.34) also describes the new data

Ratio to free nucleons FROM NEW FITS IN BLUE (In these fits, the longitudinal contribution has been assume to have no enhancement).

All three processes interfere. **TE/MEC in the deuteron**



MEC process exists for a simple deuteron, it should also exists in a heavy nucleus in which there are many two nucleon pairs which form quasi-deuterons.

process (b) is referred to as the MEC process process (c) is referred to as Isobar excitation.

e.g. Δ++ has a magnetic moment about twice that of the proton (2.7) or neutron (-1.9). So the magnetic form factor of the Δ++ -->Δ++ is 4 times that of of P-->P

If the contribution from virtual isobar excitation (c) to TE is large, then it is reasonable to parameterize TE as larger effective magnetic form factor of the bound nucleon (since the Δ++ is almost purely transverse)

$$G_{Mp}^{nuclear}(Q^2) = G_{Mp}(Q^2) \times \sqrt{1 + AQ^2 e^{-Q^2/B}}$$
$$G_{Mn}^{nuclear}(Q^2) = G_{Mn}(Q^2) \times \sqrt{1 + AQ^2 e^{-Q^2/B}}.$$

(*Note: Unlike electron scattering which is dominated by longitudinal response function at low* $\boldsymbol{Q^2}$*, neutrino cross section is dominated by the transverse part even at low* $\boldsymbol{Q^2}$*)*

We now investigated what this parameterization predicts for neutrino scattering. This model has no free parameters.

We parameterize TE in a nucleus as a larger effective magnetic form factor of the bound nucleon.

$$G_{Mp}^{nuclear}(Q^2) = G_{Mp}(Q^2) \times \sqrt{1 + AQ^2 e^{-Q^2/B}}$$
$$G_{Mn}^{nuclear}(Q^2) = G_{Mn}(Q^2) \times \sqrt{1 + AQ^2 e^{-Q^2/B}}.$$

This prescription assumes that there is no enhancement in longitudinal scattering, or in the axial contribution in neutrino scattering.

Longitudinal (L) – scattering from charge. *Charge is conserved, Coulomb sum rule is found to be valid in electron scattering. Since no enhancement is seen in the longitudinal scattering it implies that the charge distribution of bound nucleons is not changed in a nucleus.*

***Transverse (T) – Scattering from currents*, orbital angular momentum and Dirac and anomalous magnetic moments**. *These are not conserved (e.g. Meson exchange currents)*

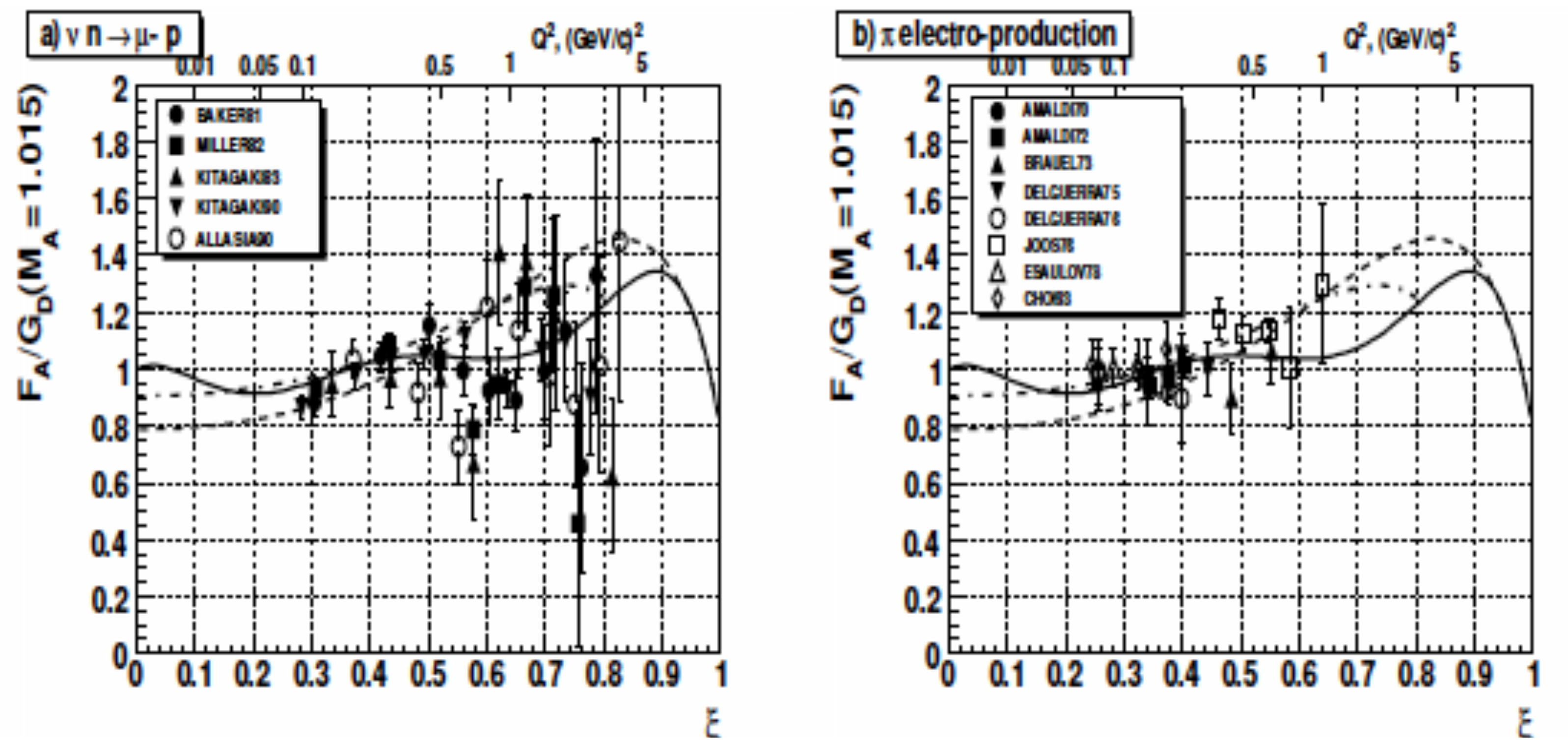
Axial current is partially conserved, *so we assume that axial form factor is not modified in a nucleus.*

The above prescription implies that the vector amplitudes from MEC/TEC interfere with axial current in the non-TE Transverse component

Predicting neutrino QE cross sections on nuclear target from electron scattering data

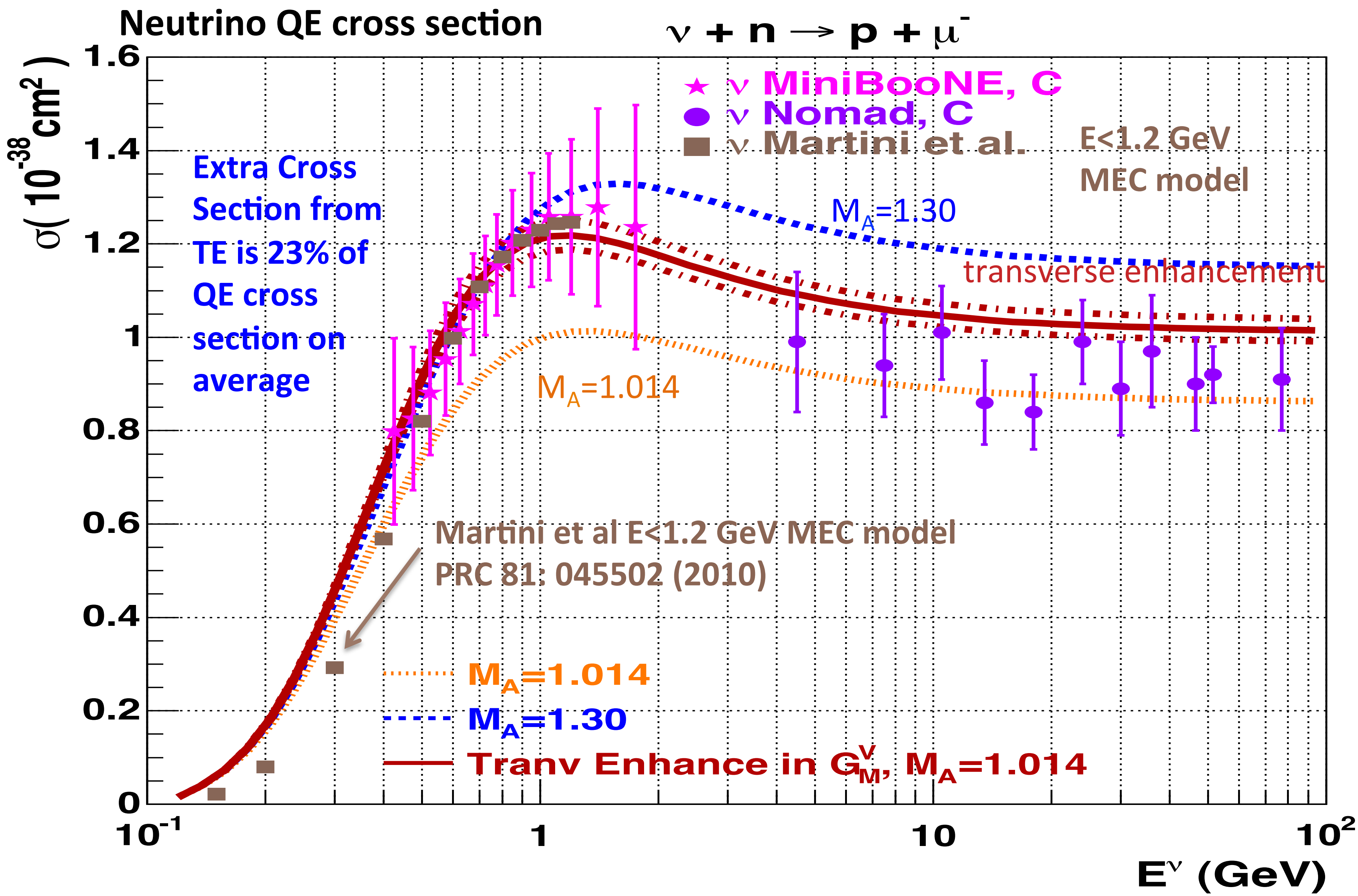
- Use free nucleon vector form factors.
- Use free nucleon axial form factor (M_A=1.014 GeV)
- Model transverse enhancement as an increase in the magnetic form factor of bound nucleons (e.g. from additional currents, but the source is not really relevant in this approach).

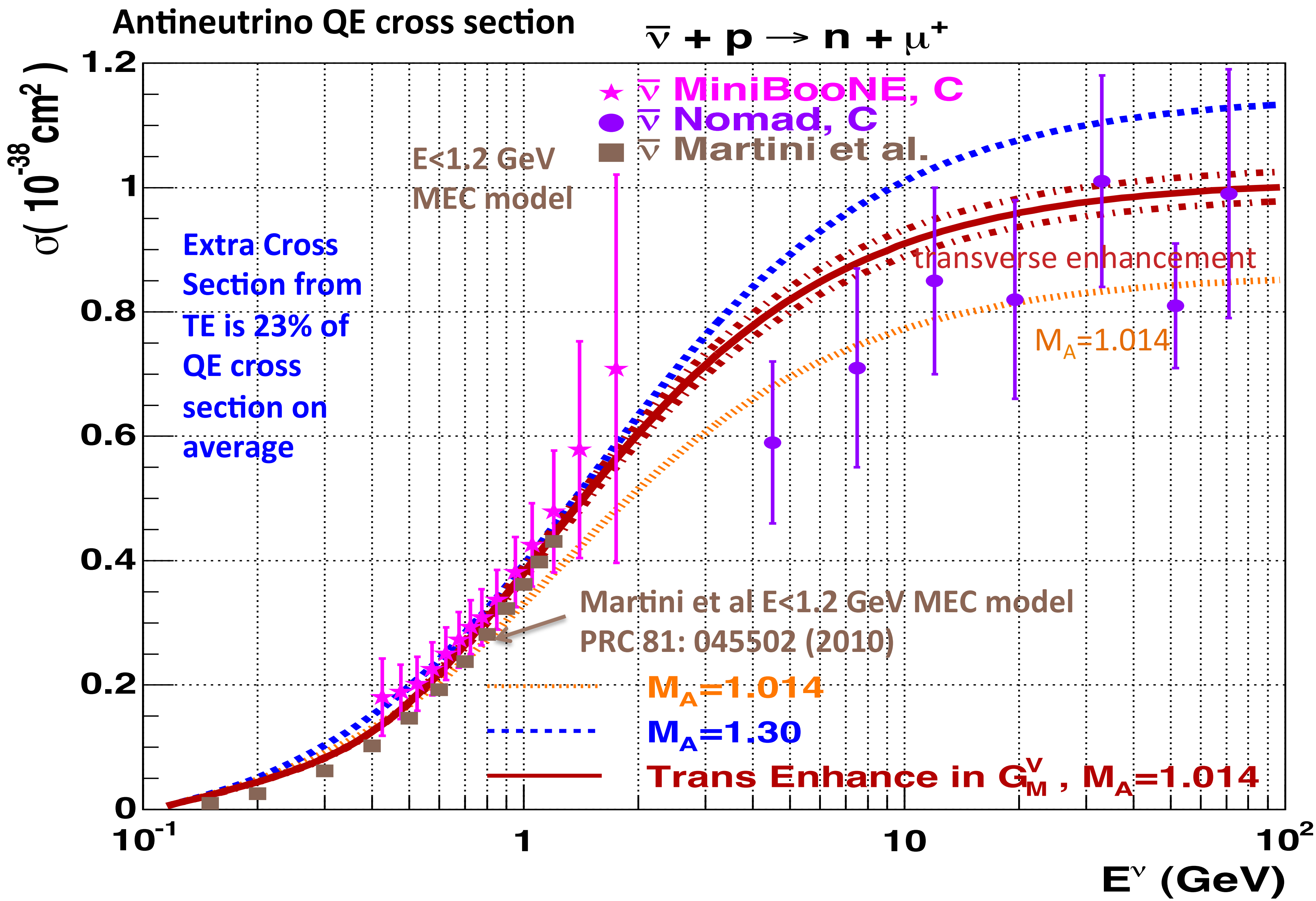
What is the axial dipole mass and how well does it fit data on free nucleons



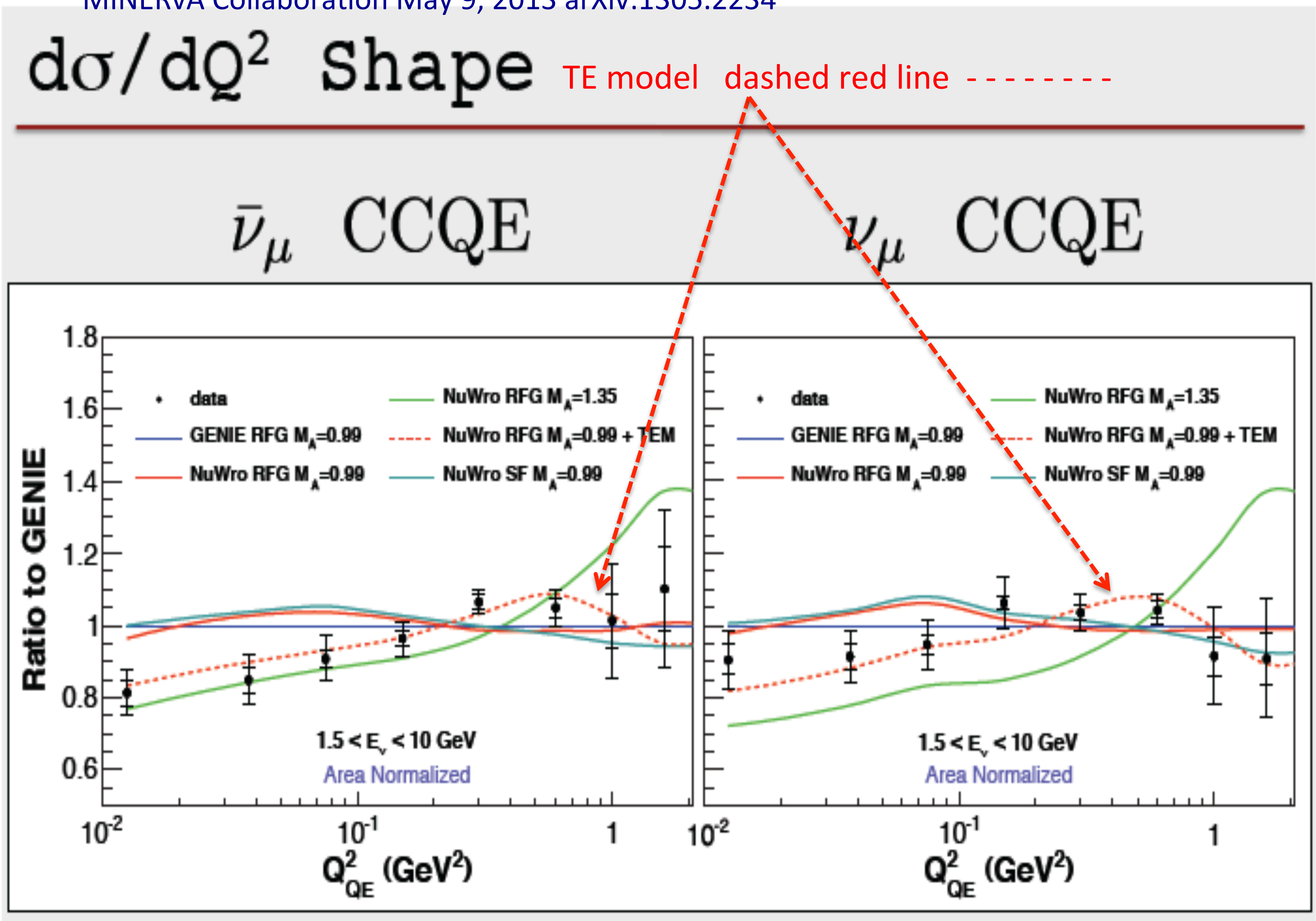
Bodek et al (BBBA), Eur. Phys. J. C53, 349 (2008). ***M_A= 1.014 ± 0.014 GeV***. (Reanalysis of deuterium data with updated vector form factors) ***The error in M_A is small if one assumes the dipole form factor. However, none of the vector form factors are pure dipole.***

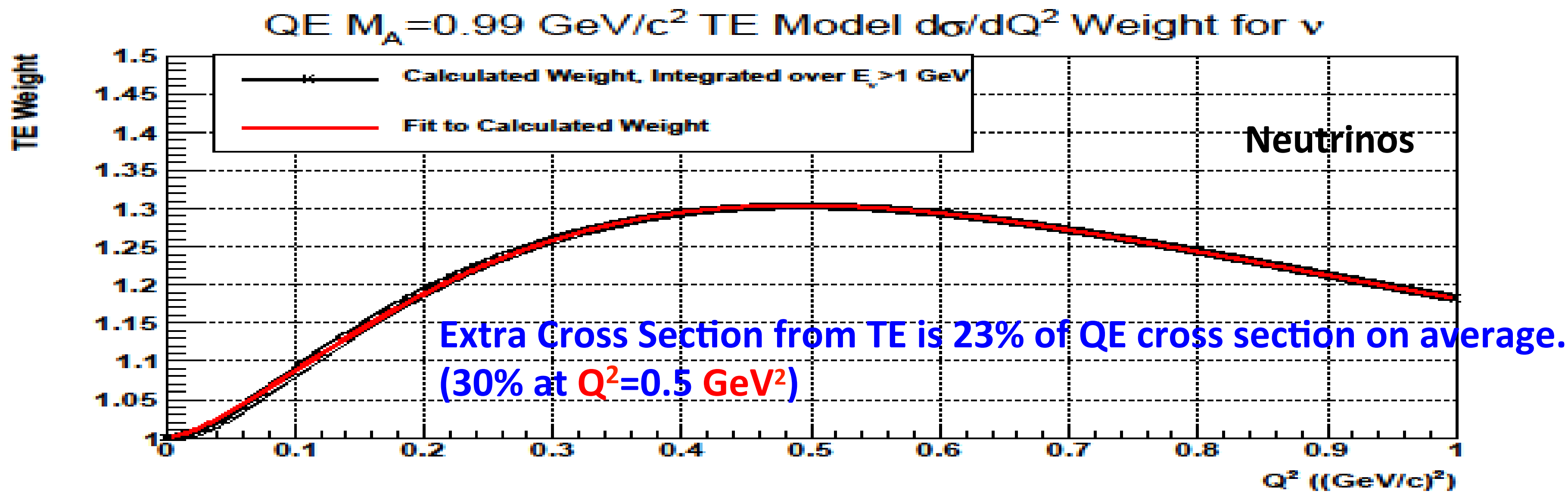
The solid line is a duality motivated **modified dipole** form that is consistent with both neutrino D and pion electro-production data. The difference between the two predictions can be used to estimate the systematic error from uncertainties in FA.


Neutrino QE cross section
ν + n → p + μ⁻
σ(10⁻³⁸cm²)
ν MiniBooNE, C
ν Nomad, C
ν Martini et al.
E<1.2 GeV
MEC model
Extra Cross
Section from
TE is 23% of
QE cross
section on
average
MA=1.30
transverse enhancement
MA=1.014
Martini et al E<1.2 GeV MEC model
PRC 81: 045502 (2010)
MA=1.014
MA=1.30
Tranv Enhance in GMV, MA=1.014
Eν (GeV)


Antineutrino QE cross section
$\bar{\nu} + p \rightarrow n + \mu^+$
$\sigma(10^{-38} cm^2)$
$\bar{\nu}$ MiniBooNE, C
$\bar{\nu}$ Nomad, C
$\bar{\nu}$ Martini et al.
E<1.2 GeV
MEC model
Extra Cross
Section from
TE is 23% of
QE cross
section on
average
transverse enhancement
M_A=1.014
Martini et al E<1.2 GeV MEC model
PRC 81: 045502 (2010)
M_A=1.014
M_A=1.30
Trans Enhance in G_M^V , M_A=1.014
E^ν (GeV)

1. Measurement of Muon Neutrino Quasi-Elastic Scattering on a Hydrocarbon Target at *Eν*~3.5 GeV MINERvA Collaboration . May 9, 2013 e-Print: arXiv:1305.2243
2. Measurement of Muon Antineutrino Quasi-Elastic Scattering on a Hydrocarbon Target at *Eν*~3.5 GeV MINERvA Collaboration May 9, 2013 arXiv:1305.2234





Ratio of neutrino QE $d\sigma_{QE}/dQ^2$ with and without TE.

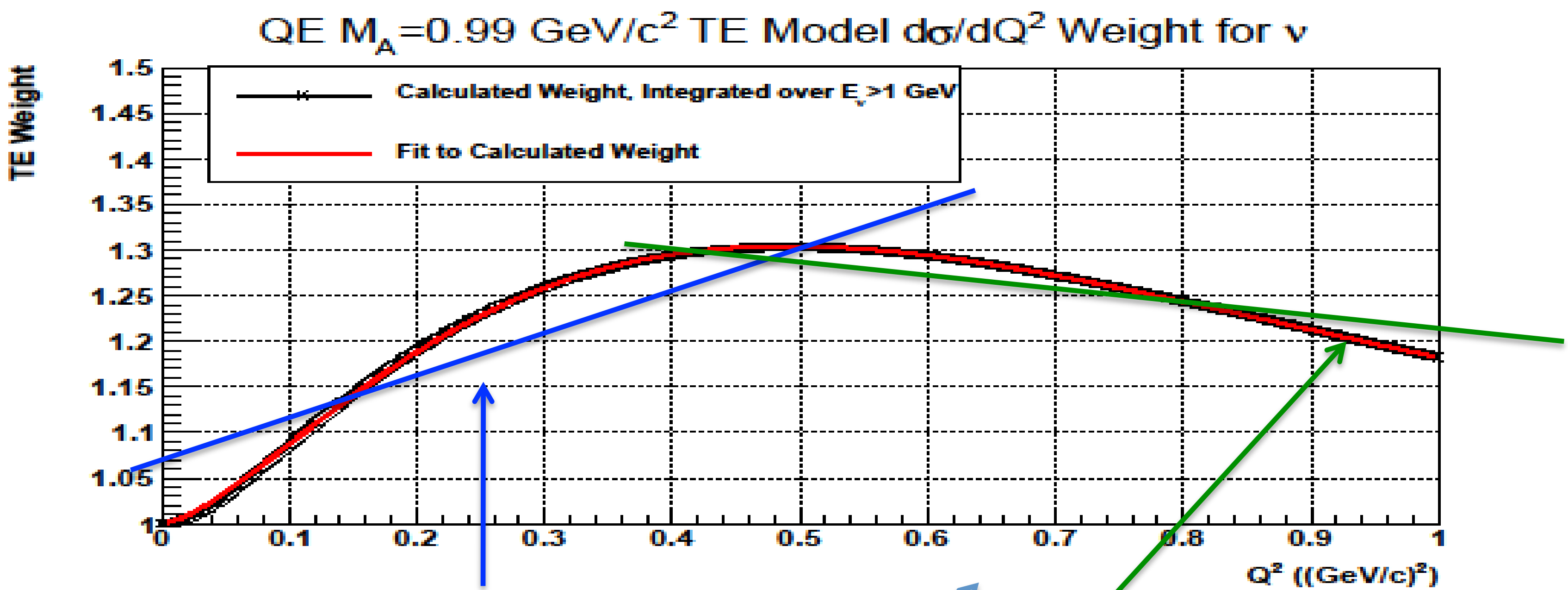
For neutrino energies greater than 1 GeV, the same function describes both neutrinos and antineutrinos (Functional form below is from Ulascan Sarica BS Thesis U of R, 2013).
We can use this functional form to weight GENIE QE events to include TE (this requires no change in GENIE).

$$R_{\nu}^{QE-TE} = 1 + \left[4.51156 \cdot \left(Q^2\right)^{1.57538} \cdot exp\left(-3.20978 \cdot Q^2\right)\right]$$
$$R_{\bar{\nu}}^{QE-TE} = 1 + \left[4.52711 \cdot \left(Q^2\right)^{1.57751} \cdot exp\left(-3.21362 \cdot Q^2\right)\right] \qquad (2.3)$$

This weighting include the effect of TE on average, it accounts for the increase in the total cross section, and for the change in shape of the Q^2 distribution. However, it will not account for possible difference in shape in n (hadron energy) for QE and TE

Why MiniBooNE finds a large MA while Higher energy experiments find a smaller MA.

If you include TE, all experiments should get MA=1. What if TE is not included?



MiniBoone has a low Q2 max, can only fit low Q2. Get MA>1 since the don't include TE

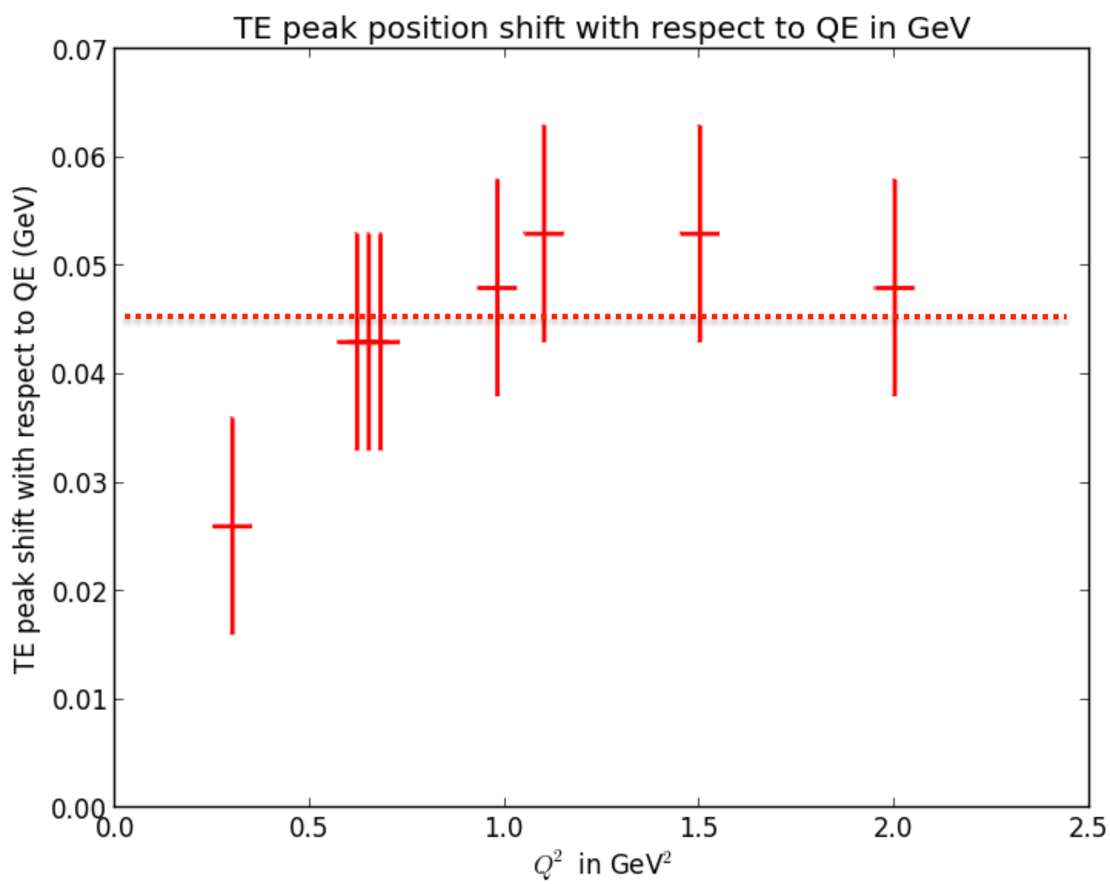
High energy experiments remove low Q2 data from fit. Get MA<1 since they don't include TE

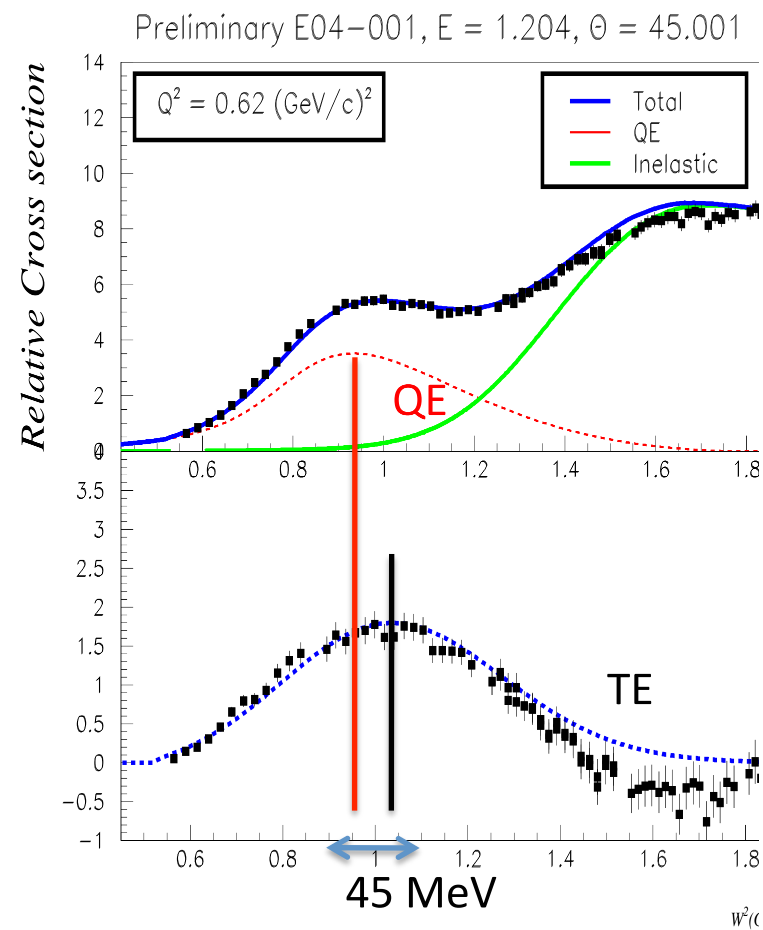
Investigation of peak and width of TE

- Modeling TE as an effective increase in the magnetic form factor of bound nucleons assumes that the QE independent nucleon component and the TE/ME component have the same shape in final state W (or equivalently energy transfer ν).

- Therefore, we now compare the shape of the QE and TE components.

Comparison of peak position of TE and QE





- Difference is 45 MeV.
- TE peak is about 45 MeV higher in ν than the independent nucleon QE peak.

RMS width of the ν distribution
For QE scattering with Fermi momentum k

Simple derivation:

QE scattering with Fermi motion k. $W^2=M^2$

$W^2=M^2 + 2M\nu - 2k*q - Q^2$ --→ $\nu = Q^2/2M + k*q/M$

$\langle\nu\rangle_{RMS} = \langle k*q_3/M\rangle = Q_3\langle k_3\rangle/M$

With $Q_3 = \mathrm{sqrt}\,\{Q^2 (1+ Q^2/4M^2)\}$

expect RMS increases with q3 with a slope of K_3/M

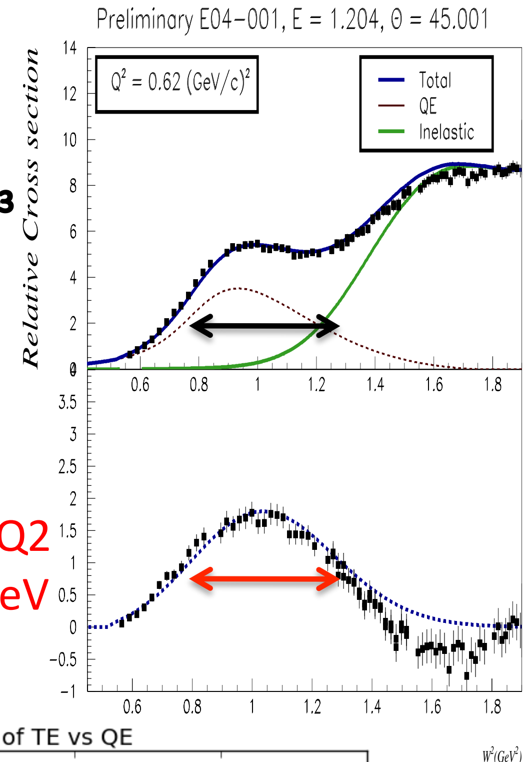
Here k_3 is the Fermi momentum along Q_3 which is the 3-momentum transfer to the nucleon.

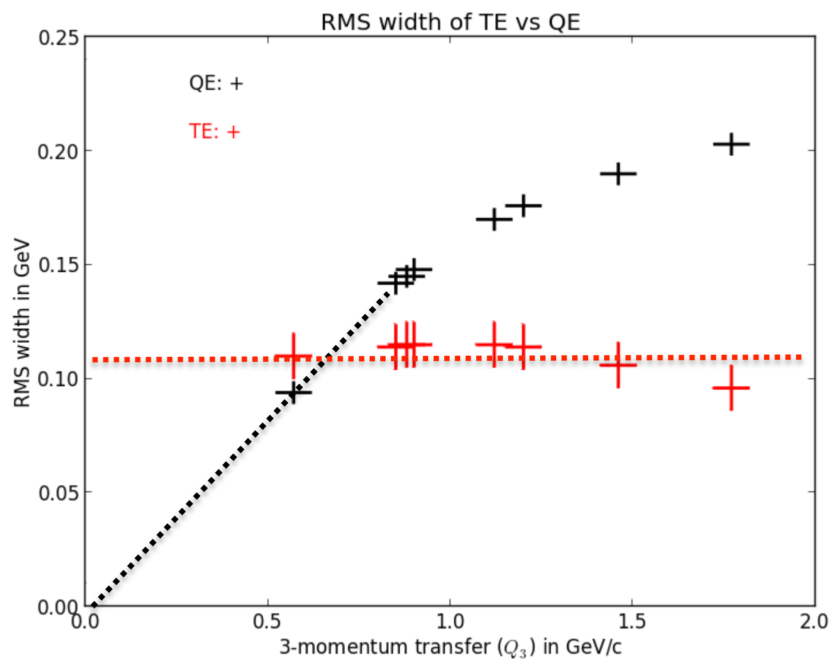
RMS width of QE rising with Q_3

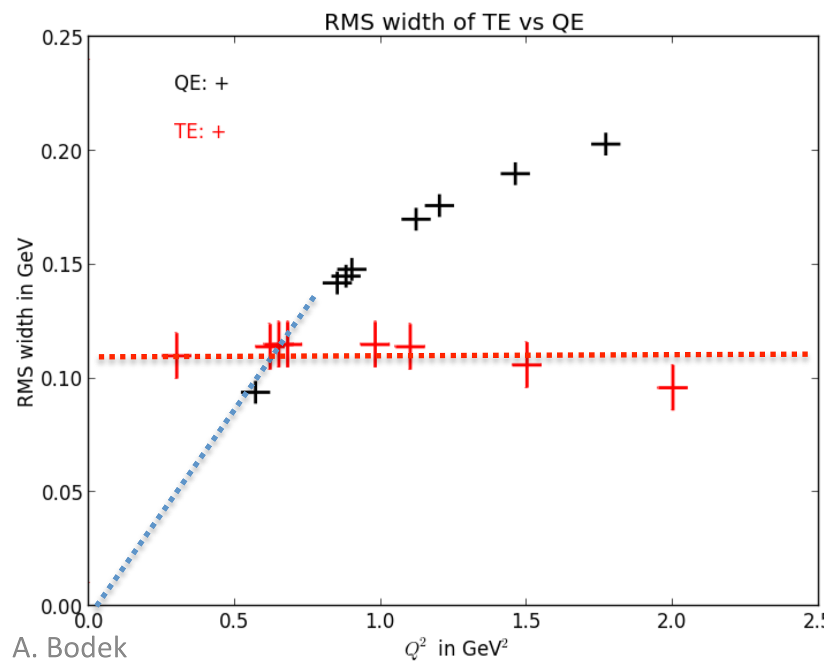
RMS ~ 0.15 GeV x Q_3

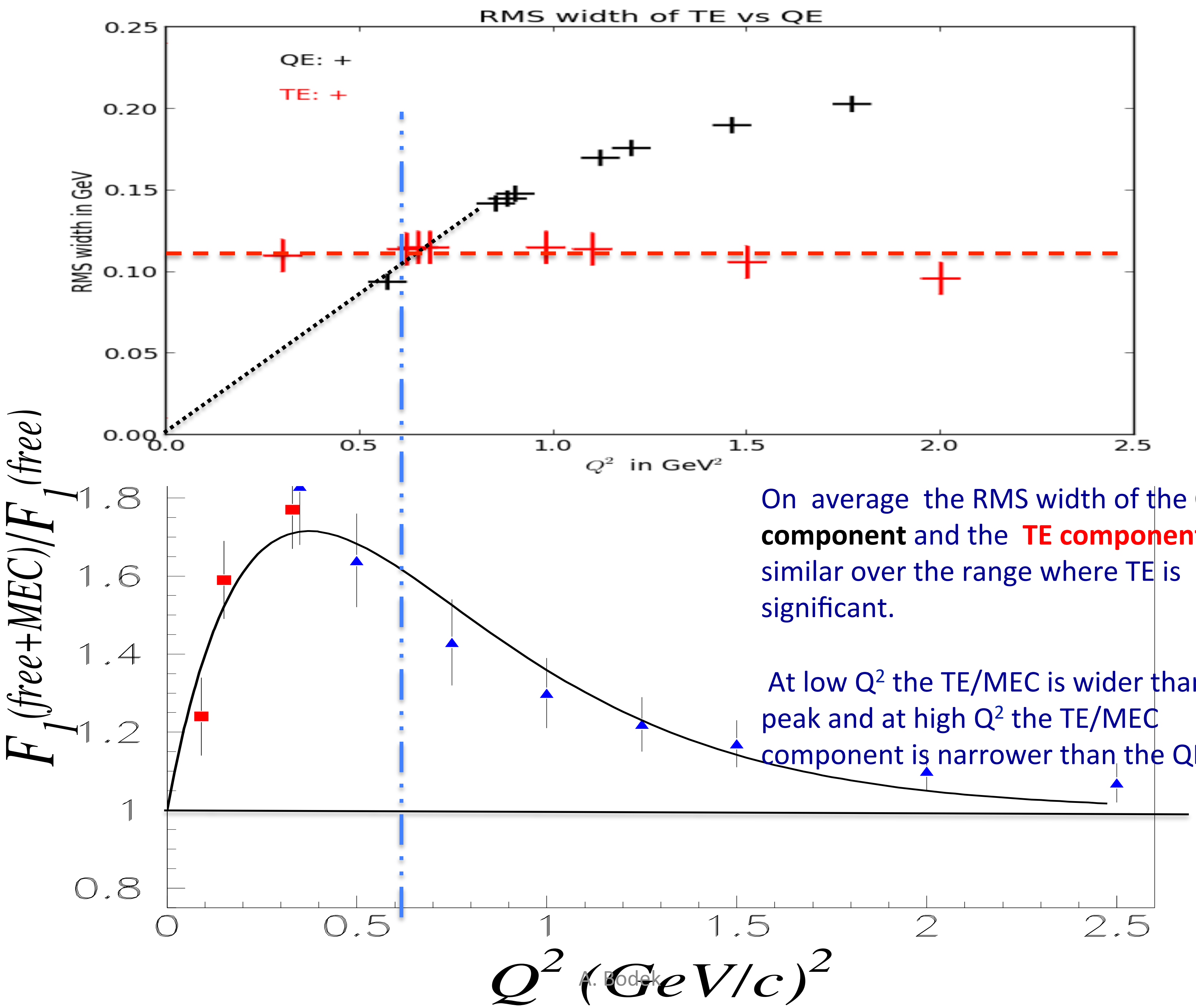
RMS width TE independent of Q2

$\langle\nu\rangle_{RMS} =$ 0.11 GeV









On average the RMS width of the **QE component** and the **TE components** are similar over the range where TE is significant.

At low Q^2 the TE/MEC is wider than the QE peak and at high Q^2 the TE/MEC component is narrower than the QE peak.

Conclusions on TE

- We have updated the analysis of the Q^2 dependence of TE. The updated analysis has smaller error bars and yields somewhat lower TE contribution vs Q^2. Although we have a new parameterization, the original parameterization still describes the new data reasonably well.

Updated parameterization A= 5.19 and B= 0.376

$$\mathcal{R}_T = 1 + AQ^2 e^{-Q^2/B}$$

- TE increases the QE cross section and changes the shape of $d\sigma_{QE}$. This can be included in Neutrino MC generators by a simple Q^2 dependent weight. The Q^2 dependent weight is the same for neutrinos and antineutrinos.

We also extracted the peak position and shape (width) in n for the TE as a function of Q^2 .

- The TE peaks relative to the QE peak positions are shifted by 45 MeV towards higher nu . The shifts are independent of Q^2 .
- The RMS widths of the n distribution of TE are about 110 MeV and are also independent of Q^2.

If we average over the Q^2 range where TE is significant, the TE and QE distributions are similar.

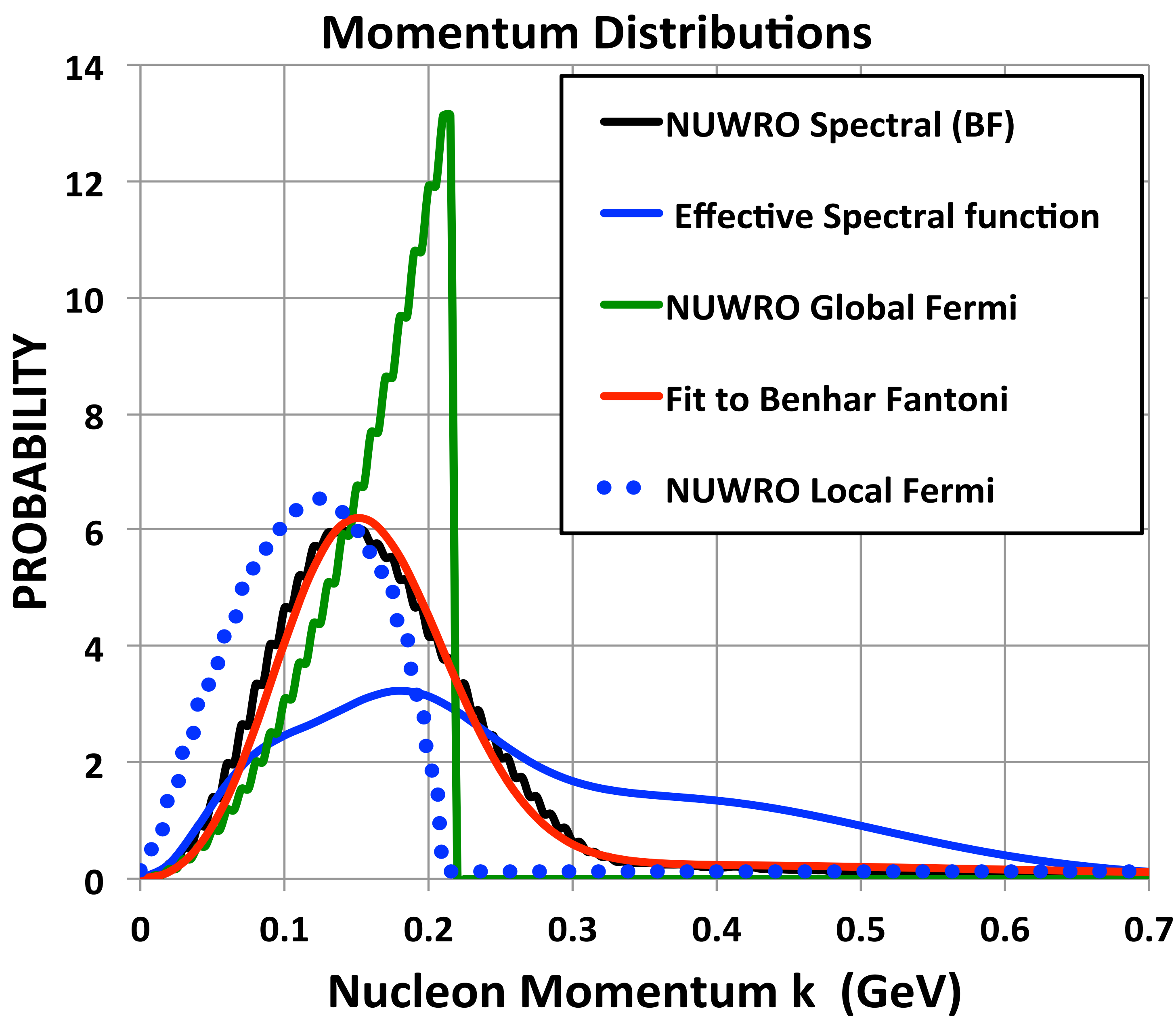
This is the reason why the simple assumption that TE can be described as increasing the effective magnetic form factors of bound nucleons works reasonably well on average.

What I have not covered

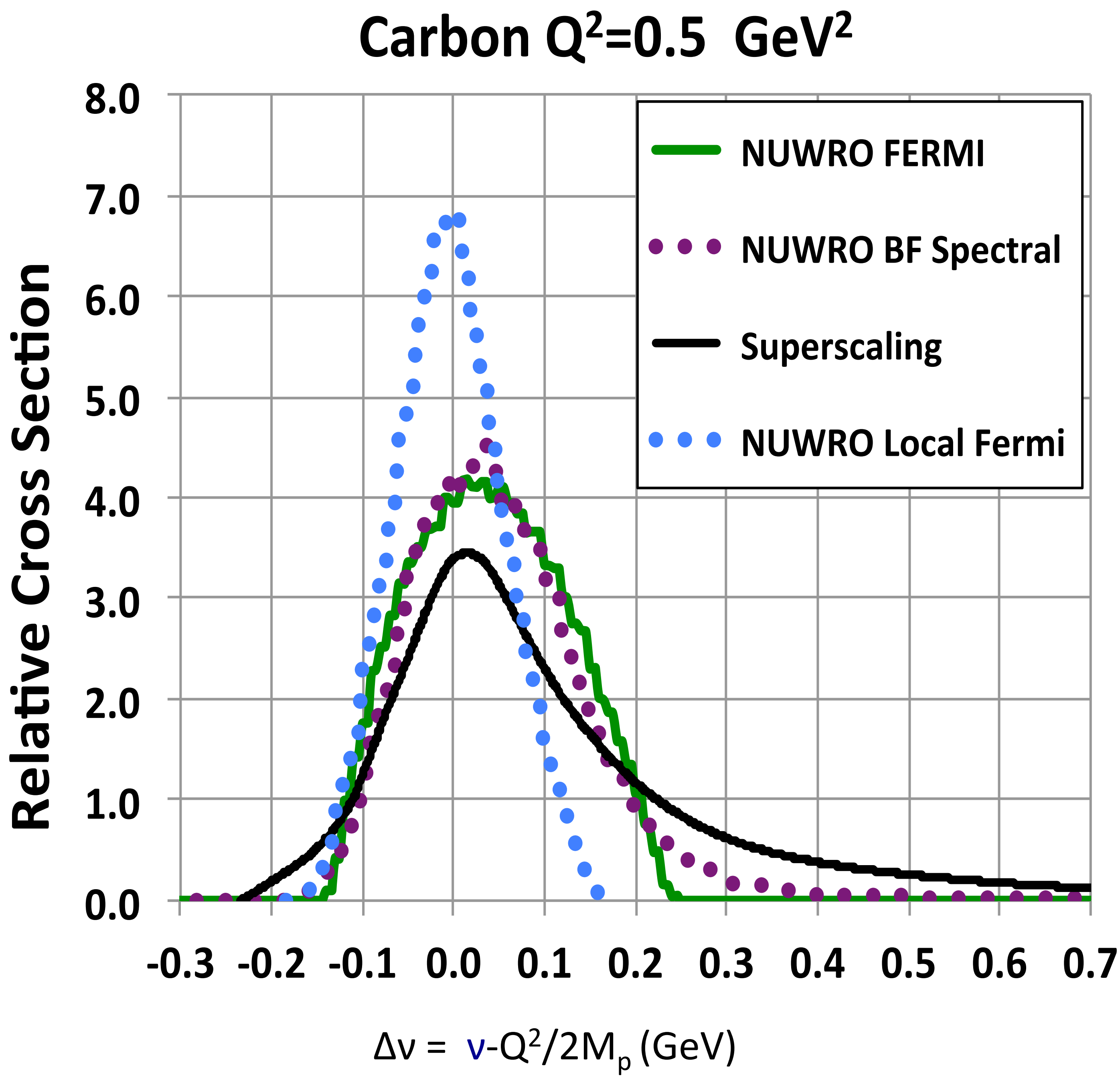
The Bodek-Yang model extracts "effective leading order Parton Distribution Functions" from electron scattering data to model neutrino cross sections in the inelastic region.

There are also ongoing efforts to update resonance production model by using recent electron scattering data in the resonance region.

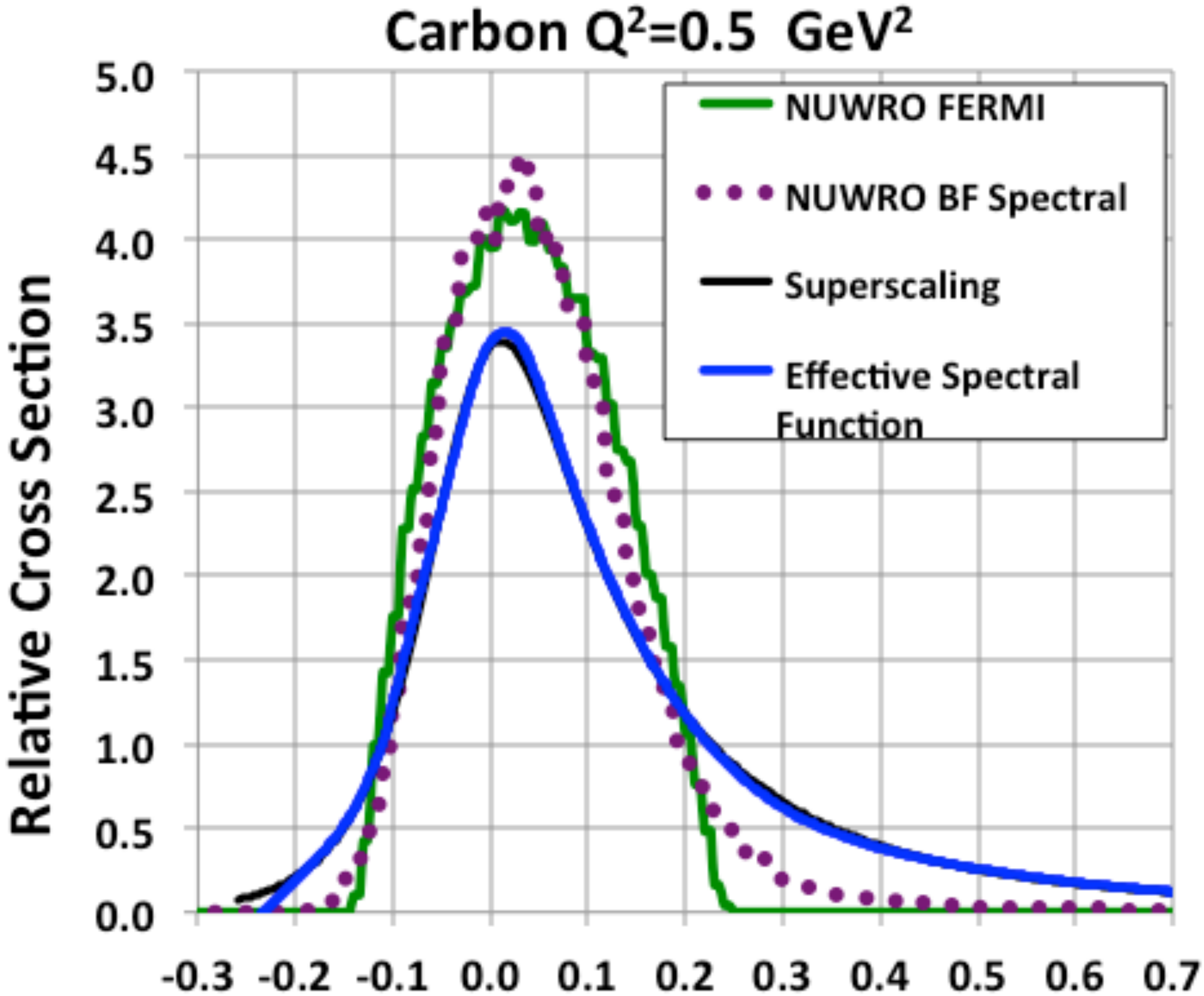
Extra Slides



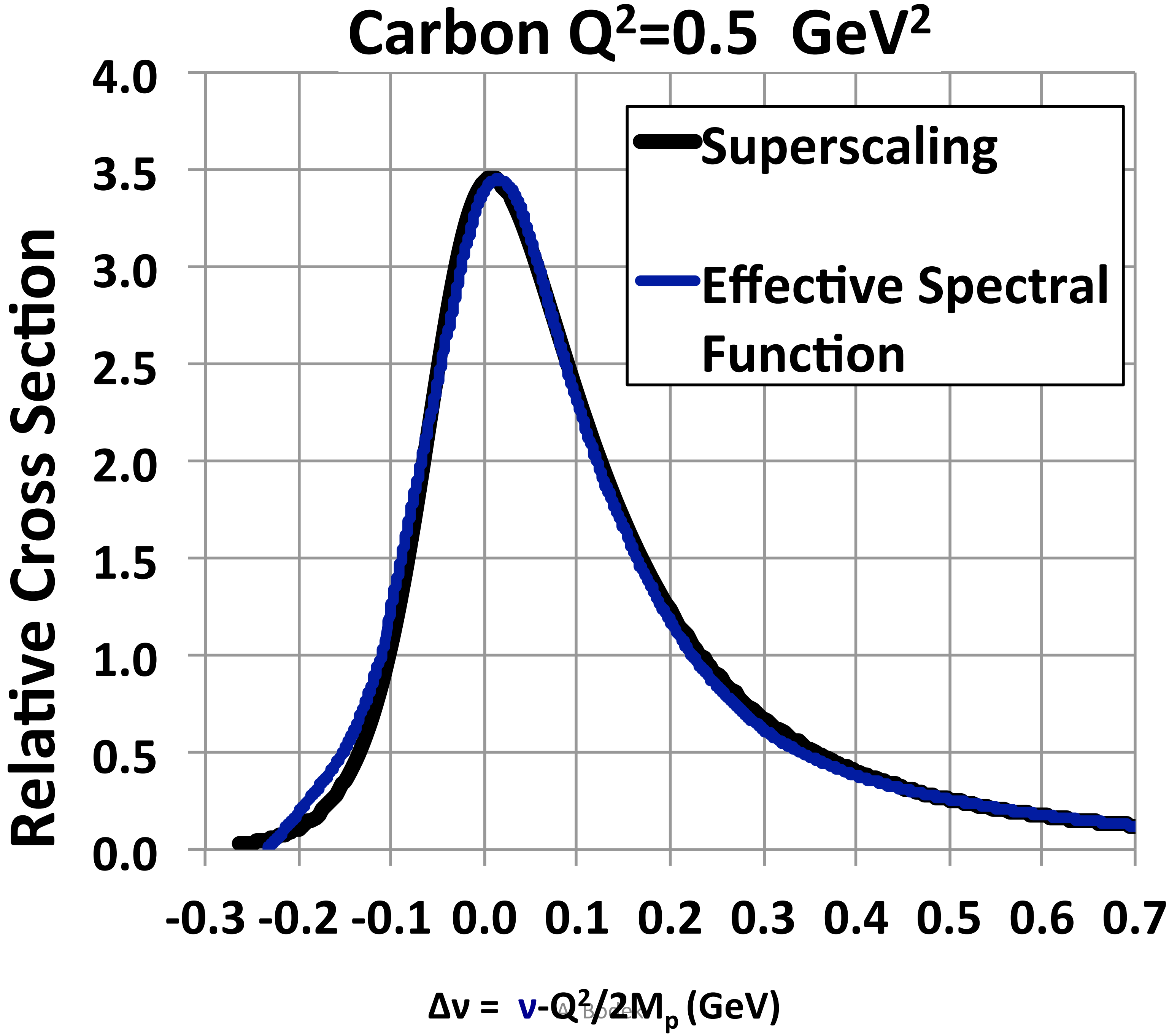
Momentum Distributions
PROBABILITY
0
2
4
6
8
10
12
14
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Nucleon Momentum k (GeV)
NUWRO Spectral (BF)
Effective Spectral function
NUWRO Global Fermi
Fit to Benhar Fantoni
NUWRO Local Fermi

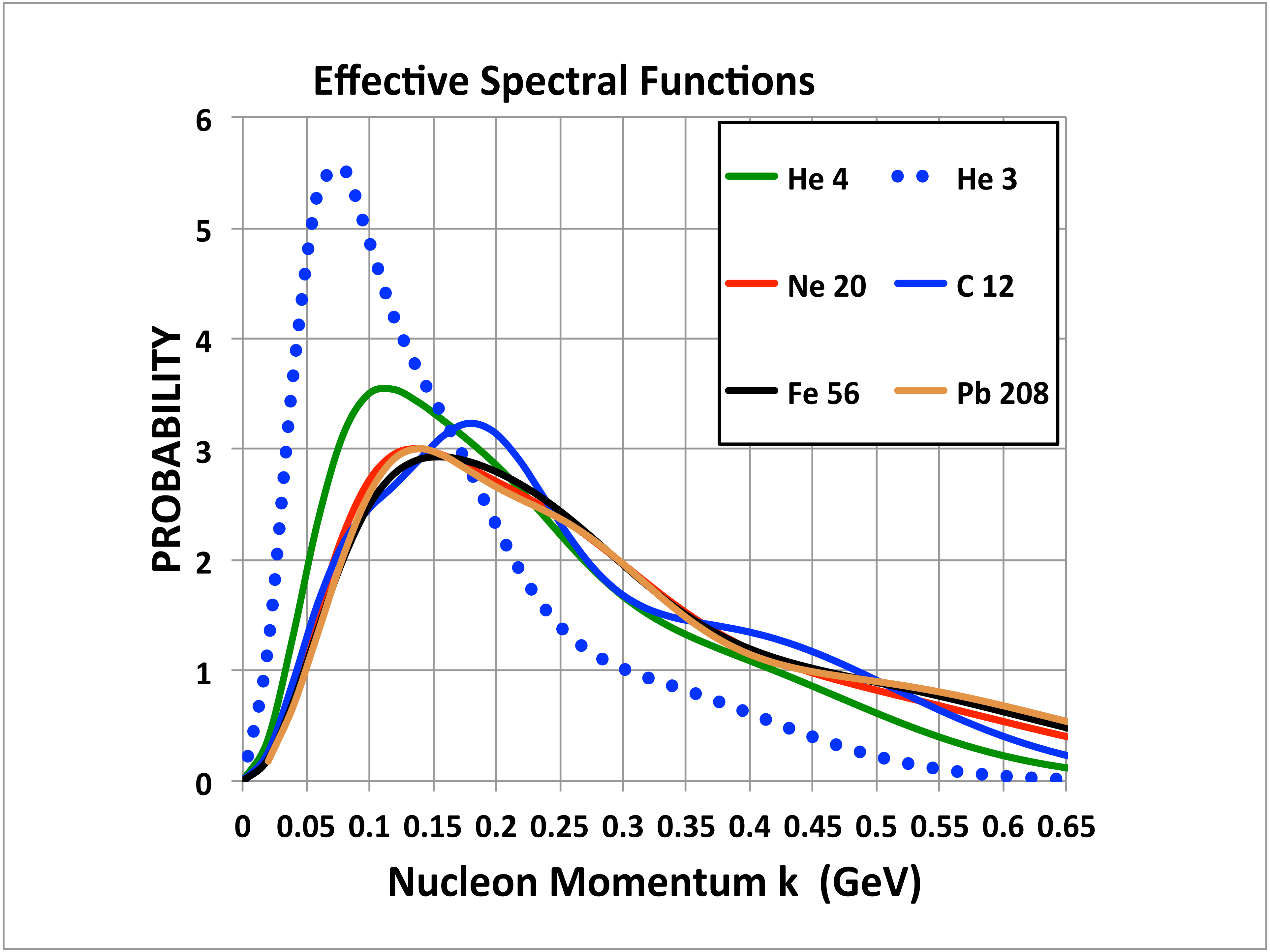


Carbon Q²=0.5 GeV²
Relative Cross Section
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
NUWRO FERMI
NUWRO BF Spectral
Superscaling
NUWRO Local Fermi
Δν = ν-Q²/2Mp (GeV)

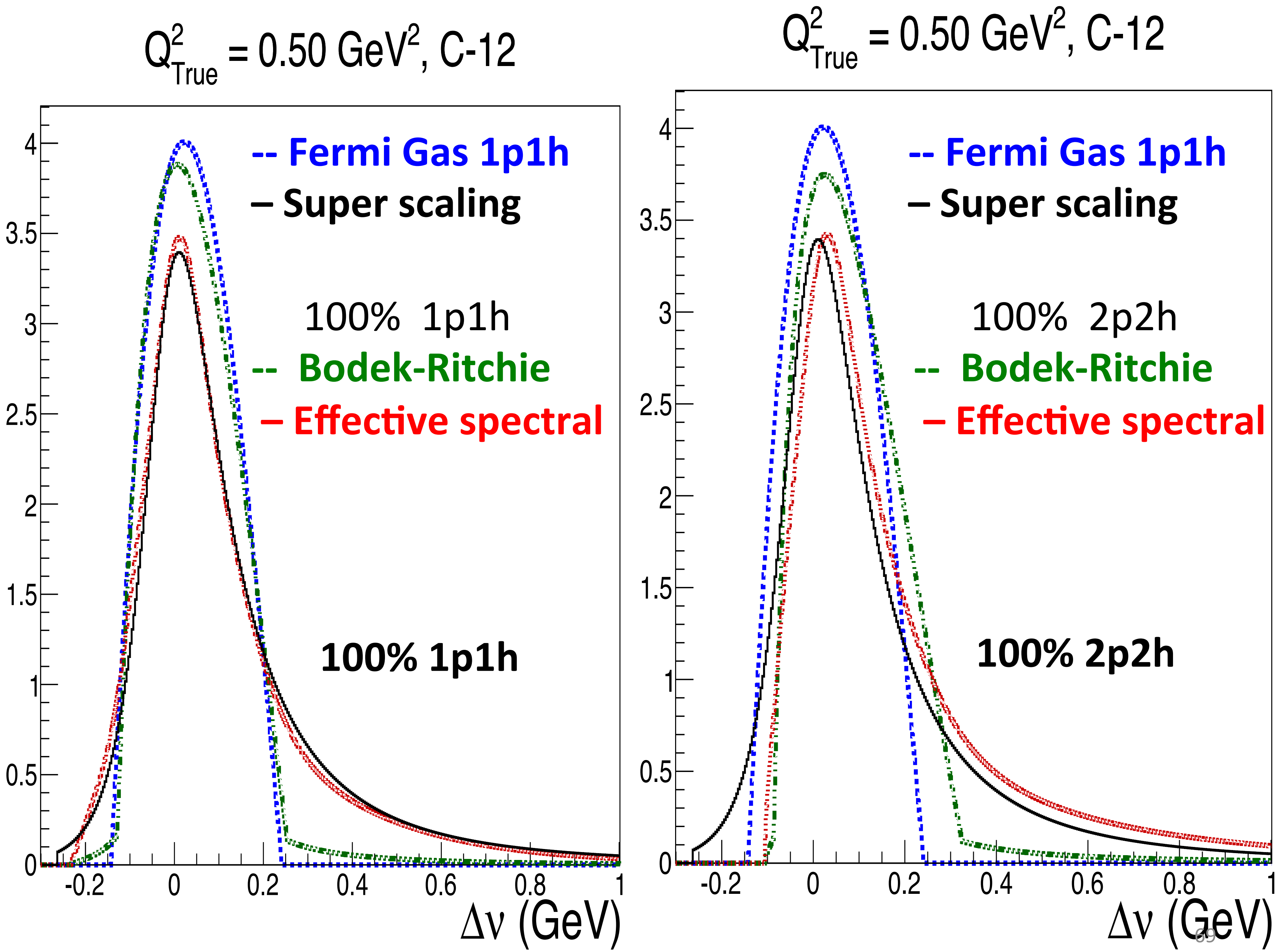


$\Delta\nu = \nu - Q^2/2M_p$ (GeV)


Carbon Q²=0.5 GeV²
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Relative Cross Section
Superscaling
Effective Spectral Function
-0.3 -0.2 -0.1 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
Δν = ν-Q²/2M_p (GeV)



Effective Spectral Functions
PROBABILITY
Nucleon Momentum k (GeV)
0
1
2
3
4
5
6
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
He 4
He 3
Ne 20
C 12
Fe 56
Pb 208


Q2True = 0.50 GeV2, C-12
-- Fermi Gas 1p1h
– Super scaling
100% 1p1h
-- Bodek-Ritchie
– Effective spectral
100% 1p1h
Δν (GeV)
Q2True = 0.50 GeV2, C-12
-- Fermi Gas 1p1h
– Super scaling
100% 2p2h
-- Bodek-Ritchie
– Effective spectral
100% 2p2h
Δν (GeV)